MECHANISTIC MODELING OF AN AQUAPONIC CONTROLLED-ENVIRONMENT AGRICULTURE SYSTEM: NUTRIENT AND WATER DYNAMICS, HARVEST PRODUCTIVITY, AND WASTE TREATMENT

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To my family and friends

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I would like to thank my parents, who raised me to appreciate and protect nature, and who instilled in me a creative ingenuity which drives me to pursue efficient solutions to life's problems. I am grateful to my dad for consistently reminding me that anything is possible when treated as a character-building exercise, and to my mom for always holding me to high standards and teaching me never to underestimate myself. I would also like to thank my mentors, advisors, and fellow graduate students at Georgia Tech for all their help and guidance in this project. I have learned so much from them and look forward to seeing this work interconnected with their respective research efforts in the future.

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LIST OF SYMBOLS AND ABBREVIATIONS

CEA	Controlled-environment agriculture system
ASM	Activated Sludge Model
CMFR	Completely mixed flow reactor
INFEWS	Innovations at the Nexus of Food, Water, and Energy Systems
$\mu_?$	Specific growth rate for an organism
$k_?$	Maximum specific substrate utilization rate for an organism
$Y_?$	Yield coefficient for an organism
$X_?$	Particulate component
$S_?$	Soluble component
$b_?$	Decay coefficient for an organism
$K_{S_{i,j}}$	Half-saturation coefficient for a specific substrate *i* for the growth of organism *j*
$K_{I,i,j}$	Haldane inhibition coefficient for a specific substrate *i* for organism or process *j*
$i_{i,j}$	Fraction by mass of component *i* in the component *j*
X_S	Particulate organic matter (slowly biodegradable substrate)
S_S	Soluble organic carbon (easily biodegradable substrate)
S_{ON}	Soluble organic nitrogen
S_{OP}	Soluble organic phosphorus
S_{SRP}	Soluble reactive phosphorus
S_{NH}	Ammonium nitrogen
S_{NO3}	Nitrate nitrogen
X_{BH}	Viable heterotrophic biomass

X_{BA}	Viable autotrophic biomass
S_{O2}	Dissolved oxygen
X_I	Inert material (nonbiodegradable)
C_{fish}	Concentration of fish biomass
PlantDensity	Plant density per square meter
S_K	Soluble potassium ion
S_{Ca}	Soluble calcium ion
S_{Mg}	Soluble magnesium ion
T_W	Water temperature
T_{GH}	Greenhouse air temperature
PPFD	Photosynthetic photon flux density
k_a	Ammonification rate coefficient
k_m	Phosphorus mineralization rate coefficient
k_La	Oxygen mass transfer coefficient to water
$S_{O_2,sat}$	Oxygen saturation concentration
k_{hyd}	Maximum specific hydrolysis rate for X_S
RH	Relative humidity
T_{out}	Temperature outside the greenhouse
P_{atm}	Atmospheric pressure
h	Approximate crop height
z_m	Height of wind and humidity measurements
u_z	Wind speed
Q_o	Water flow rate between tanks
Q_f	Water flow rate from fish tank
Q_b	Water flow rate from bioreactor

Q_p	Water flow rate from plant tank
Q_A	Airflow rate
ω_{ext}	Humidity ratio of external air
ω_{GH}	Humidity ratio of greenhouse air
ρ_{ext}	Density of external air
ρ_{GH}	Density of greenhouse air
LCA	Life cycle assessment
Δ	Slope of saturation vapor pressure curve at T_{GH}
R_n	Net radiation
G	Soil heat flux
ρ_a	Mean air density at T_{GH} and P_{atm}
c_p	Specific heat of air
e_s	Saturation vapor pressure at T_{GH}
e_a	Actual vapor pressure at T_{GH}
r_a	Aerodynamic resistance
r_s	Bulk surface resistance
γ	Psychrometric constant
λ	Latent heat of vaporization of water

SUMMARY

Aquaponics, the symbiotic co-culturing of fish and vegetable crops, is a promising technology for both food production and waste mitigation. As part of an urban ecosystem, controlled-environment-agriculture (CEA) systems would serve as nutrient transformation hubs, generating food and removing nutrient pollutants from local organic waste and wastewater. A mechanistic model of the nutrient and water dynamics of this emerging system is proposed here, based on Monod kinetics and the International Water Association's Activated Sludge Models (ASM). This model functions to dynamically predict the nutrient transformation and food production capacity of an aquaponics CEA, and allows the optimization of crop and fish species selection, planting and stocking densities, fish food composition, feeding rate, maximum harvest rates, and other important factors with important economic, policy, and design decision implications.

CHAPTER 1. INTRODUCTION

Modern industrial food production practices pose a serious threat to the environment. Of the total greenhouse gas emissions emitted by the US in 2015, 7.9% were from agricultural sources, not including life-cycle embodied carbon emissions for processing, refrigeration, or transport[1]. Currently, large-scale agricultural practices contribute to the pollution and eutrophication of surface water[2,3], deplete crucial groundwater aquifers in drought-stricken areas of California[4], contribute significantly to deforestation worldwide, and rely on extensive fertilization with inorganic fertilizers mined from rock or chemically synthesized. Food can travel many hundreds of miles before reaching consumers and is unevenly accessible, creating food deserts in rural and urban areas alike. With controlled-environment agriculture, a solution may be found to these problems while also diverting waste streams from landfills and wastewater treatment plants: food grown in urban areas where it is needed most, using local waste streams as nutrient inputs, would prevent the inefficiencies of modern industrial agriculture and enhance urban sustainability.

1.1 Embodied resources in centralized industrial agriculture

Food requires water, energy, nutrients, chemicals, and fossil fuels to be grown, protected from pests, transported, and kept fresh. Lengthy cold supply chains occur on a transcontinental or even trans-global scale. 40% of America's total supply of specialty fruits, vegetables, and other table foods, including 70% of lettuce and an even higher proportion of tomatoes, are grown in California's Central Valley and collectively require 20% of the entire nation's groundwater[5,6]. The San Joaquin and Sacramento River Basins

which supply this fertile valley lost 20.3 km^3 of groundwater storage capacity between October 2003 and March 2010[4], an unsustainable level of loss in such a critical area for national food security and sovereignty. The centralized nature of food production means it travels an average of 6,760 kilometers over its life cycle[7], a resource-intensive supply chain requiring fossil fuels for transport and constant refrigeration and which results in the loss of the food's nutritive value in some cases[8].

Fruits and greens are delicate, and typically must be treated with significant amounts of pesticide, herbicide, and fungicide, and fruiting crops need plenty of fertilizer to maintain production capacity and quality[9]. These costly inputs not only detract from farmers' profits, but contribute to pollution of surface and groundwaters. One study found that nearly 66% of nitrogen fertilizer applied is lost to the environment, with between 20 and 40% of those losses flowing directly into surface waters[3]. Excess available nutrients in waterways are the leading cause of eutrophication, in which large algal and cyanobacterial blooms deplete the dissolved oxygen concentration in water bodies, sometimes to fatal levels for fish and other aquatic wildlife[2].

Another key embodied resource in food production is the fossil fuel required for nearly every stage of the plant's life cycle: tractor operation pre- and post-harvest, processing and packaging operations, and refrigerated transport over hundreds or thousands of miles can add up to more than 500 liters of diesel fuel per hectare per crop for lettuce alone[9].

In traditional indoor aquaculture operations, fish are fed with commercial fish food made in part from fishmeal, an unsustainable resource coming from rapidly-depleting wild

fish stocks[10]. Energy requirements can be substantial: water is pumped, heated, aerated with blowers, and must be treated to remove solids and ammonia, which is another chemical- and energy-intense process. Many fish farms are not fully recirculating, and release nutrient pollution in their effluent water which contributes to eutrophication. Treating effluent to meet emission regulations can be costly and technically demanding for aquaculture operations.

1.2 Controlled-environment agriculture as a sustainable option

CEAs are specially-designed greenhouses in which crops are grown under full environmental control. These systems include careful and precise monitoring of macro- and micronutrients, humidity, light intensity, air circulation, and other factors that are important for increased plant growth. Conventional threats to productivity such as insects, competition from weeds, inclement weather, and fungal infections are mostly or completely avoided, and plants receive exact dosages of nutrients for optimal growth. These systems provide fresh, out-of-season produce and require few pesticides, fungicides, or herbicides.

Hydroponics, a soilless growth technique which can take several different design formats, is the principal method used in most CEAs: nutrients are added in precise ratios to water which is pumped or sprayed directly over plant roots and then recirculated within the system. In the floating raft method, roots are supported by small amounts of inert media

and submerged in aerated water containing dissolved nutrients. This model is mainly concerned with the floating raft culture method (see Figure 1).

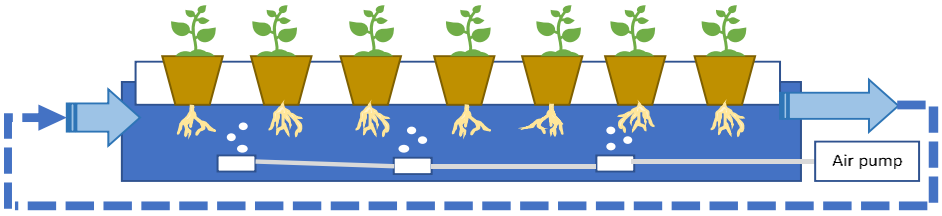

Figure 1. Floating-raft hydroponic culture schematic

In a natural ecosystem, animal wastes and decayed organic matter are the source of all the fertilizer used by plants. Drawing on this basic concept, aquaponics is an extension of the hydroponic technique which combines aquaculture with hydroponic vegetable growth, using the nutrients from fish wastes to fertilize hydroponic crops. Bacteria transform the wastes into more bioavailable inorganic forms, and the plants take up the dissolved nutrients before the water is recycled back to the aquaculture tanks (see Figure 2). In these systems, the nutrient input comes from fish food rather than commercial hydroponic nutrient solutions or minerals.

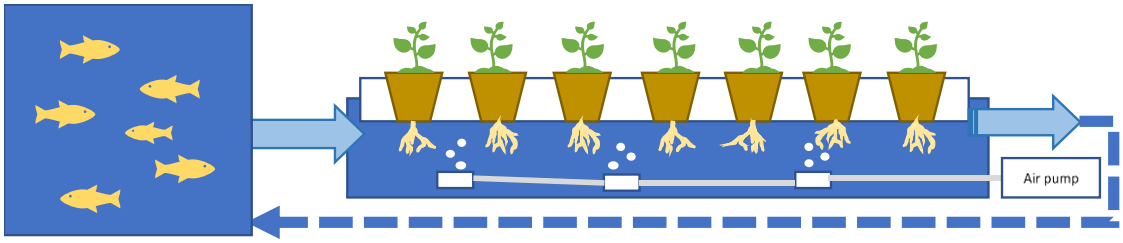

Figure 2. Floating-raft aquaponics system general schematic

1.3 CEA efficiency

Due to the ability to contain and recycle water within these systems, hydroponic systems use up to 66 times less water than conventional farms[6] and can produce much higher yields per unit area than traditional farms. Conventional lettuce grown in California can be harvested only one to three times per year, while soilless systems allow continuous harvest, resulting in 29 and 10 times higher lettuce yields per unit area for hydroponic and aquaponic CEAs respectively[6]. Plants grow quickly in these systems because nutrients are delivered in bioavailable forms directly to their roots at the optimal concentrations and ratios, and because light levels, temperature, humidity, and other potential stressors are perfectly controlled or removed. While these systems are not practical for use with staple grain crops such as corn, wheat, soybeans, or rice, they have great potential as high-efficiency growth methods for leafy greens, herbs, berries, tomatoes, peppers, squash, and other specialty fruits and vegetables which provide vitamins and minerals essential for human health and balanced nutrition.

A fully-controlled environment requires a significant amount of energy, however, which can negate some of the benefits of CEA production and presents a sustainability problem for this technology. One comparative study found that hydroponic vegetables require 30 times more energy than conventional California-grown ones[6]. Additionally, in vegetable-only hydroponic systems, nutrient solutions contain the same inorganic components used in conventional farming, which are either artificially synthesized or acquired through mining rock such as apatite and evaporite (potash)[11]. Apatite, which contains the phosphate used in fertilizers, is a finite resource[12]. Nitrogen fertilizer, in the form of ammonia, is synthesized using the Haber-Bosch process and is especially energy-intense, requiring about 1,100 m^3 of natural gas per metric ton of anhydrous ammonia

produced[13]. Thus, the continued use of these fertilizers in a CEA would not represent a significant increase in energy sustainability over conventional agriculture.

1.4 INFEWS and interconnected urban infrastructure systems

Current practices are unsustainable, as climate change, peak phosphorus, and peak oil continue to threaten the status quo. Large-scale sustainability issues such as those posed by the current industrial agriculture system have inspired researchers worldwide to find ways to combine industries into urban ecological networks, forming productive, symbiotic linkages to reduce the need for such resource-intense processes. At Georgia Tech, Innovations at the Nexus of Food, Water, and Energy Systems (INFEWS) is an interdisciplinary, international research effort in which a complex meta-model is being constructed to address these issues with the wider goal of closing resource loops in cities by bridging gaps between industries whose waste and input streams are compatible.

CEAs could become integral in this process, increasing urban sustainability by bringing together food, water, and energy networks in a synergistic and productive way (Figure 3). The CEA would serve as a sink for organic waste materials and, potentially, wastewater, while acting as a source of fresh fish, fruits, and vegetables. This would prevent large amounts of methane-producing wastes from entering landfills and would lessen the burden on municipal wastewater treatment plants. Algae grown on the nutrients in municipal wastewater can be fed to fish and serve as the primary nutrient input to the system, for the dual purpose of increasing the nutritive value of the fish by concentrating their omega-3 fatty acid content[14] and to act as an intermediate between waste streams and the food supply. Other intermediates such as black soldier fly larvae or vermiculture

composting could also turn wastes into valuable proteins and high-quality food sources for cultured fish[15,16]. The relatively high energy demand of the CEA could be addressed through renewable energy and anaerobic digestion of compost and wastewater prior to their use as a nutrient input. Anaerobic digestion produces methane, which can be used to heat the digester itself as well as power lights, pumps, water heaters, and air conditioners. Additionally, siting the CEA near an existing power plant would facilitate the reuse of waste heat and emitted carbon dioxide, which could warm and enrich the greenhouse environment.

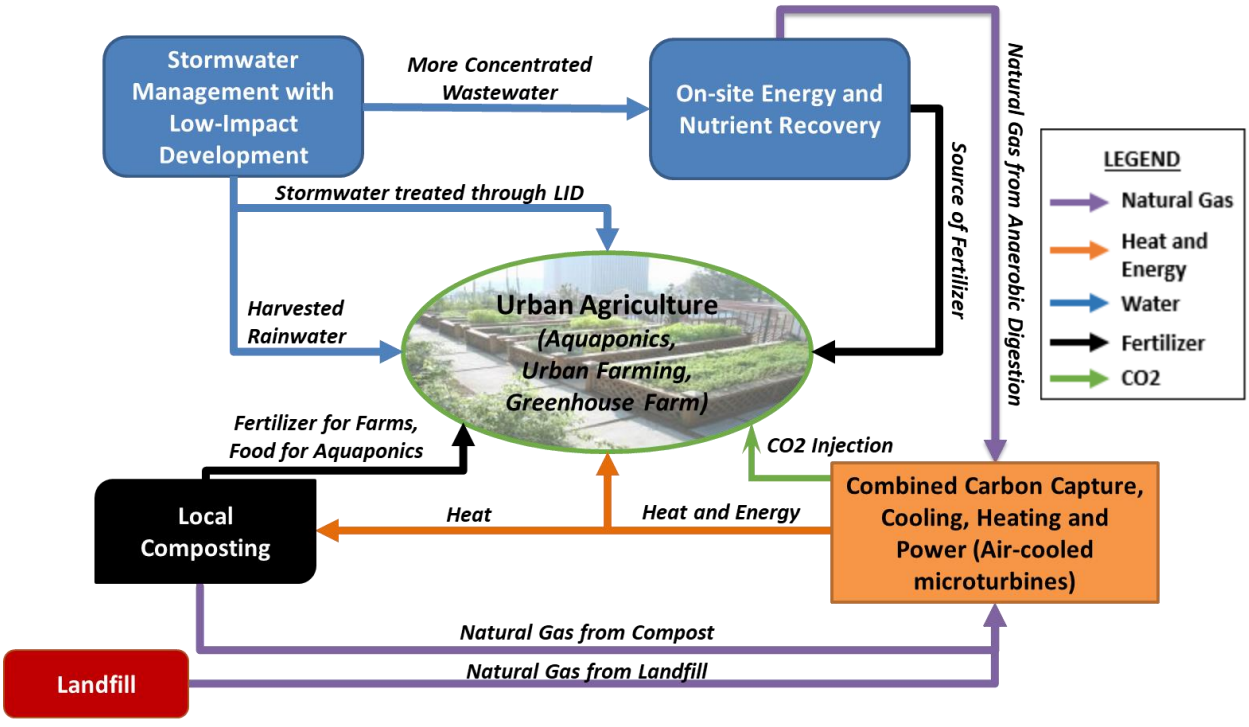

Figure 3. Overall interconnected infrastructure network schematic.[17]

Linking together new and existing urban infrastructures in this way would allow productive recycling of water, nutrient, and energy resources to retain local resources and reuse waste. Implementing these interconnected urban ecosystem networks would decentralize some food production, alleviating many of the environmental stresses caused by the current industrial food production methods by reducing the need for agrochemicals, inorganic fertilizers, irrigation water, and fossil fuel. This would have the added benefit of increasing the self-sufficiency and resilience of individual communities to local or distant crop failures due to climate-related or other natural or human-caused disasters. In the face of an uncertain climactic future, the urban CEA could provide the key to a protected, decentralized method of food production based on local nutrient and water resources.

1.5 Mechanistic modeling effort

Optimization and modeling of CEA systems has traditionally been empirical, with recommendations for nutrient input and fish-to-plant ratios based on observed 'black box' input-to-output studies[18,19]. The multifunctionality and efficiency of aquaponics CEAs makes understanding them from a basic, mechanistic level very important in their future as a large-scale infrastructure. The underlying mechanisms of the aquaponic CEA must be modeled dynamically such that practical predictions about harvest rate, fish stocking capacity and feeding rate, water and aeration demands, and nutrient removal capacity can be made prior to installation and throughout operation.

The dynamic water balance is equally important in making design and operation decisions: the water demand of these systems is not negligible and represents a limited resource, especially in arid or otherwise infertile regions where this type of agriculture is

more likely to be needed. Many urbanized areas do not currently allocate significant portions of their fresh water supply for nearby agriculture, so it is important to determine the new demand on the city's water infrastructure and at the same time determine whether rainwater capture would be sufficient to supply the demand.

The model presented here describes many of the most important biological, chemical, and physical dynamic changes that occur within an aquaponics CEA. Based on wastewater treatment models for activated sludge published by the International Water Association[20], this model describes kinetic expressions for the emission, transformation, and uptake of several key macronutrients by bacteria, fish, and plants. A dynamic water balance will allow predictions of water demand depending on outdoor climactic conditions, and can be used to moderate ventilation controls to maintain ideal greenhouse humidity.

With most of the parameters dependent on the specific system design and scale, this model can be applied to any size CEA, climate, latitude, fish or plant species, or hydroponic technique. Sophisticated predictions of outcomes can be made under dynamic conditions. The system could be optimized for maximum food production, lowest carbon footprint during construction and use, or maximum nutrient removal from waste material inputs. Experiments to increase the specificity of model parameters and add further detail to the model can be carried out at the Georgia Tech aquaponic testbed system.

The network of large-scale technologies proposed by INFEWS must be highly predictable if it is to be successful. The scale on which the systems are implemented must also be carefully calibrated to avoid over- or under-loading the system. Starting from first scientific principles, this model is an early step toward the goal of making decentralized

aquaponics a realizable and cost-effective technology for improving sustainability in urban infrastructure.

1.6 Kinetic basis of the model

This model was based on the widely-used Activated Sludge Models for wastewater treatment, making use of Monod kinetics, which state that limiting substrates will determine the growth of an organism or population of microorganisms[21]. As the organism or microbial population is exposed to increasing concentrations of its required substrates, its growth rate will increase up to a certain maximum at which point the substrate will become 'saturated' with respect to the response of the growth rate. Past this saturation concentration, further increases in substrate concentration yield negligible increases in growth rate.

The concentration of the substrate, S, at which the organism grows at half of its maximum rate is called the 'half-saturation coefficient' denoted K_S. This parameter is an important indicator used to describe the sensitivity of growth rate to the concentration of that substrate[21]. The higher this coefficient, the more of the substrate is required for growth rate to reach saturation, and the slower the growth kinetics overall. The lower the coefficient, the less of the substrate is required to reach saturation and the faster the growth rate responds to increases in that nutrient.

The form of the Monod equation used in this model is:

$$\mu = kY\left[\frac{S}{K_S + S}\right]C - bC \tag{1.1}$$

Where μ is the growth rate of a given organism [biomass (volume-time)$^{-1}$], k is its maximum specific substrate utilization rate [mass substrate used (biomass gained - time)$^{-1}$], Y is the yield coefficient [biomass gained (mass substrate used)$^{-1}$], S is the concentration of some limiting substrate [mass of substrate (volume)$^{-1}$], K_S is the half-saturation coefficient [mass of substrate (volume)$^{-1}$], C is the concentration of the organism in the reactor [mass (volume)$^{-1}$], and b is the endogenous respiration rate [time^{-1}].

If a substrate is inhibitory to growth, increases in its concentration will cause decreases in growth rate, and equation (2) is used instead of the traditional Monod equation[21]:

$$\mu = kY\left[\frac{K_S}{K_S + S}\right]C - bC \tag{1.2}$$

When more than one substrate is potentially limiting, the extended Monod equation is used, with terms that can be expanded as shown here:

$$\mu = kY\left[\frac{S_1}{K_{S_1} + S_1}\right]\left[\frac{S_2}{K_{S_2} + S_2}\right]\left[\frac{S_3}{K_{S_3} + S_3}\right]\ldots\left[\frac{S_n}{K_{S_n} + S_n}\right]C - bC \tag{1.3}$$

Where n is the number of limiting substrates, denoted $S_{1\rightarrow n}$, and including any inhibitory terms (as in Eqn. 1.2) where biologically relevant.

The combination of saturation curves resulting from each limiting nutrient describes the overall growth kinetics of the organism, and allows careful monitoring and optimization of each required input for growth as was included in this model. The ‘substrates’ used in

this model are the required nutrients or gases required for the growth of each contributing organism.

When a substrate is necessary for growth but inhibitory at too high a concentration, the Haldane function[22] is used to describe its effect on growth:

$$\mu = \frac{kYS}{K_S + S + \frac{S^2}{K_I}} C - bC \tag{1.4}$$

Where K_I is the Haldane inhibition coefficient [mass of substrate (volume)$^{-1}$]. Higher values of K_I indicate a substance with a milder inhibitory effect and vice versa[22]. This is useful for describing the kinetics of plant and fish growth in response to environmental parameters such as light intensity.

CHAPTER 2. MODEL PROCESSES

2.1 **Biological processes**

The most important processes in an aquaponic CEA are carried out by living organisms: bacteria, fish, and plants interacting symbiotically.

2.1.1 Fish

The fish tank is an ideal conceptual starting point, as it is the point of entry for the only nutrient input to the system, namely fish food. This model considers the overall mass of fish in the tank, rather than the individual fish; this simplifies calculations and removes the need to consider the different requirements and growth rates of fish at different life cycle stages. With future expansions, the model should be adapted to treat fish of different ages and sizes separately as this will be important for maximizing profit and accuracy of predictions.

Fish receive all their nutrition from their food, the composition of which is under the grower's control. Thus, the input values for macronutrient fractions in fish food ($i_{C,food}$, $i_{N,food}$, $i_{P,food}$, $i_{K,food}$, $i_{Ca,food}$, and $i_{Mg,food}$) are treated as initial conditions in the model rather than variables, and are based on established aquaculture principles for dietary needs of the fish species. This allows easy alteration of the composition of fish food to include algal meal, black soldierfly larvae, or other products of organic municipal solid waste composing such as worms from vermiculture operations. Each nutrient is assumed to follow Monod kinetics in fish metabolism, reaching saturation at some percentage of the total mass of food. This model neglects micronutrients, any nutrient interactions upon

ingestion, and potential toxicity of excessively high concentrations of any specific nutrient. The major processes carried out by fish include the addition of particulate and soluble organic wastes to the water, and the removal of dissolved oxygen. The particulate wastes include organic nitrogen, phosphorus, carbon, calcium, potassium, and magnesium entrapped within particulate carbonaceous BOD (X_S), along with an inert fraction. The initial soluble waste is ammonium, emitted through the gills. Fish were assumed not to excrete urea, as it makes up a much smaller fraction of their total nitrogenous waste[23].

2.1.2 Heterotrophs

Heterotrophic bacteria carry out several vital processes in the functioning of the CEA, oxidizing the wastes added to the water by the fish. The first important process is the enzymatic hydrolysis of particulates into soluble organic nutrients by heterotrophic bacteria. This creates readily biodegradable substrates for other heterotrophs, which use them as a source of carbon and energy to grow. Separate hydrolysis steps were modeled for the breakdown of entrapped organic nitrogen and particulate organic phosphorus at rates proportional to the amount of N or P present in the feces, respectively. Hydrolysis processes release soluble organic carbon, nitrogen, and phosphorus. Other species of heterotrophs carry out ammonification, to turn organic nitrogen into ammonia. Still others mineralize organic phosphorus into soluble reactive phosphorus (here considered to be the orthophosphate species PO_4^{3-}, HPO_4^{2-}, and $H_2PO_4^-$). Although several other types of phosphorus exist, no others were included here. A small fraction of fish feces is nonbiodegradable, and remains in the system as inert mass. Fish feces particulate matter also releases soluble Ca^{2+}, Mg^{2+}, and K^+ (S_{Ca}, $S_{Mg,}$ and S_K) at rates proportional to the fraction of each ion in the particulate matter. Bacteria are not assumed to take up significant

quantities of any of these dissolved ions. Only aerobic heterotrophic processes were included in this model.

2.1.3 Autotrophs

Autotrophic nitrifying bacteria remove ammonia and produce nitrate as a waste product. Though this process occurs over two kinetic steps with nitrite as an intermediate, it was simplified into one step in this model as was done in the ASM models[20]. Nitrite was not modeled as a state variable and was assumed to be present in very low concentrations. No anaerobic or anoxic autotrophic processes were included, notably denitrification.

2.1.3.1 Bacterial decay

When bacteria die, they become particulate carbonaceous BOD, which is then hydrolyzed by other heterotrophs into soluble nutrients.

2.1.4 Plants

After the bacteria transform organic nutrients from fish wastes into inorganic, soluble forms, they are available for absorption by plant roots. Plants take up the inorganic forms of nitrogen (nitrate and ammonia), with the optimal ratio between these two forms strongly dependent on plant species as well as environmental and soil content conditions[24,25] For lettuce, nitrate is the preferred form of nitrogen, with additions of ammonium-N reducing growth rate[26]. As a simplification of a complex plant growth response to the ammonium:nitrate ratio and due to modeling problems caused by very low ammonium-N, the plant growth kinetic equation was altered such that plants only absorb nitrate-N.

Plants take up phosphorus in the form of soluble reactive phosphorus. No other forms are considered available to plants.

Potassium is often a limiting nutrient in aquaponic systems[27], as fish also require significant amounts and do not usually excrete adequate amounts in their waste for plants to thrive. This model does not consider the addition of any external source of potassium, but it can be modified to include this. Instead, the model in its current state could be used to predict how much to add and when, in case of deficiencies. Potassium does not form metal-ion complexes easily, and thus 100% of dissolved K^+ ions are assumed available to plants. An excess of potassium was included in this model for ease of viewing kinetic relationships and to prevent inhibition of growth of either fish or plants.

Magnesium and calcium are also required by plants in small amounts, and are considered macronutrients. These are absorbed by plant roots when they are in their free ionic form.

Micronutrients were not considered in this iteration of the model due to the complexity of including the many interactions between micro- and macronutrients in overall plant nutrition. Their concentrations are also extremely small in solution and plants do not require large amounts of these elements to grow.

2.2 Nonbiological processes and physical controls

2.2.1 Aeration

Aeration was assumed to be in excess in the CEA according to the GT testbed design: all tank types are heavily aerated with submerged air stones. However, the

minimum required aeration level could be determined by the model equations and would be a useful parameter for later energy demand calculations. Fish require a minimum critical dissolved oxygen concentration to survive, and above this level their dependence on oxygen follows approximate Monod kinetics until reaching a saturation point[28]. For tilapia, which can survive in low-oxygen water by using atmospheric oxygen, the minimum dissolved oxygen threshold is about 1mg/L.[29] Plant roots require more dissolved oxygen for cellular respiration to occur, with half-saturation growth rate occurring around 5mg/L.[30] This is because, as non-aquatic plants, roots are not adapted to being permanently submerged in water. The oxygen mass transfer coefficient was assumed to be quite high for this system, since the assumption in this iteration was that dissolved oxygen would be in excess to better visualize the existing kinetic relationships.

2.2.2 Solids removal

The method used here for breaking down biodegradable solids is an aerated bioreactor (CMFR design). Bacteria in this tank hydrolyze the solids into soluble organic and then inorganic compounds, which are subsequently available to plants in the next tank. A sludge wasting term was not added in this iteration to better facilitate an understanding of the heterotrophic capacity for solubilizing particulates into useful forms for plants; in an ideal system with a very efficient bioreactor, few solids would need to be wasted since the majority would be hydrolysed and removed by plants. Here, nonbiodegradable solids tend to build up in the system water, as there is no removal mechanism. In later versions of the model, sludge wasting will be incorporated to more accurately represent the mass that must be removed from the system.

2.2.3 Water temperature, unionized ammonia, and BOD

Fish are additionally sensitive to other environmental qualities including BOD, water temperature, and unionized ammonia (NH_3) concentration. Elevated BOD is toxic to fish, following an inhibitory curve with increasing S_S concentration above a certain critical value. Monod kinetics were assumed. Haldane temperature dependence was used for fish growth as a translation of the relationships seen in the literature[31] which indicate an ideal temperature range with both extremes detrimental to growth rate. However, in troubleshooting the code, the function appeared to consistently and significantly inhibit fish growth and was ignored for this iteration with the assumption of ideal water temperature instead. Unionized ammonia dependence was not modeled here, as the impact on growth rate is only seen at concentrations above a certain critical value[28]. Because of the nature of the efficient bioreactor, high levels of aeration, and relatively neutral pH in the system, the NH_4^+:NH_3 ratio was assumed $\approx$ 1000:1 or more[32].

2.2.4 Light intensity, greenhouse temperature, humidity, and P_{CO2}

For plants, the list of environmental dependencies is longer. Light intensity must be great enough to allow maximum growth rate, but too much light can slow growth as chloroplasts migrate to the vertical edges of the cells to avoid photodamage[33]. Photosynthetic photon flux density (PPFD) (µmol photons $(m^2\text{-day})^{-1}$), represents the flux of photons within photosynthetically active wavelengths onto a given area over a given period of time. Haldane kinetics were assumed for plant light intensity dependence and temperature dependence.

Crops are similarly sensitive to humidity levels: when too low, stomata must close to prevent excessive water vapor loss, and in doing so they cannot take in carbon dioxide and photosynthesis slows[34]. When humidity is too high, the rate of nutrient uptake is depressed: the smaller the difference between the water vapor concentration in the air spaces of the leaf and that of the surrounding atmosphere, the slower the rate of evaporation from the leaf and therefore the slower the upward movement of solutes into the roots[34]. Low rates of nutrient uptake result in slower growth. Humidity was assumed held constant at 70% by automated ventilation controls.

Atmospheric carbon dioxide dependence also follows Monod kinetics for many plant species, reaching saturation at levels above those in the natural atmosphere, around 600ppm.[35]

2.2.5 Evaporation, evapotranspiration, ventilation, and water replacement

Evaporation from the exposed water surface of the fish tank causes a net transfer of water into the atmosphere. This process was approximated as a constant based on historical pan evaporation rates in Atlanta[36]. Due to the Styrofoam rafts covering the plant tanks, evaporation is neglected there, and no water losses are modeled for the bioreactor.

Evapotranspiration rate is dependent on numerous environmental factors including light intensity and duration, wind speed, temperature, relative humidity, and leaf area. The Penman-Monteith evapotranspiration function was used to model this process[37].

Water vapor leaving the tanks by evaporation and evapotranspiration is lost from the system boundary via the vents in the greenhouse walls and roof.

When fish and plant material is harvested, the water embodied within them is removed from the system. Each organism was assumed to contain a fixed percentage of water by mass. The water embodied within fish fry, eggs, and plant seeds and seedlings was assumed to be negligible. All water replacements are composed of pure water or treated rainwater with no dissolved nutrients or minerals.

2.3 Process schematics

The processes described in the preceding section occur in different sectors of the CEA. Bacterial processes take place in all three tanks. Plant and fish-specific processes are restricted to rafts and fish tanks respectively.

2.3.1 Fish tank schematic

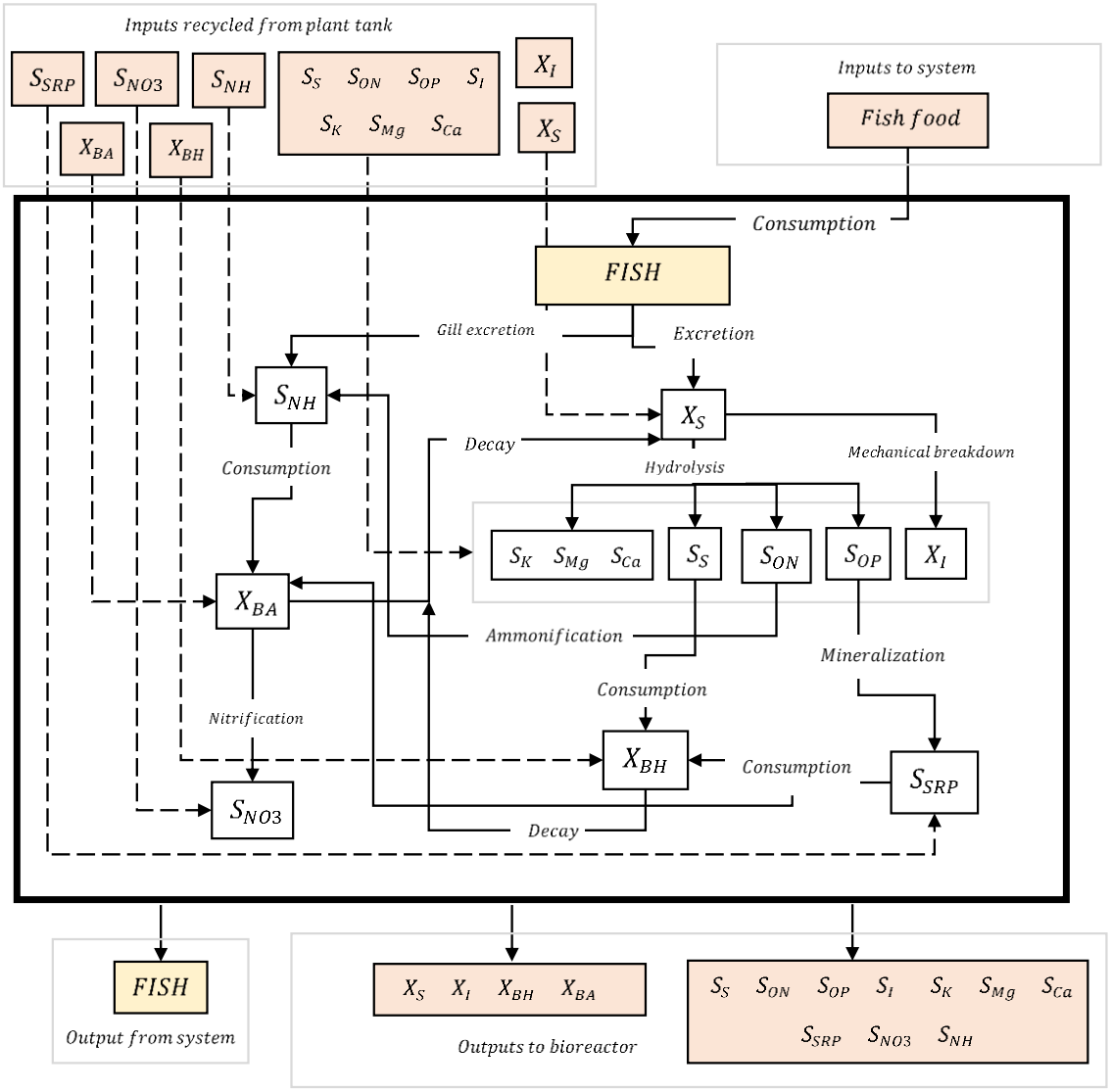

Figure 4. Schematic process diagram of the modeled fish tank system

As fish add nutrients to the water in both soluble and particulate forms, bacteria hydrolyze and transform them into soluble and inorganic forms according to the schematic above. The major particulate waste X_S encompasses carbon, nitrogen, phosphorus, and inert fractions, which are hydrolyzed at rates proportional to their relative mass percentages in the X_S mixture, which is composed of both fish feces and dead bacterial mass. Inputs from the plant tank recycle back into the fish tank and are shown as inputs.

2.3.2 Bioreactor schematic

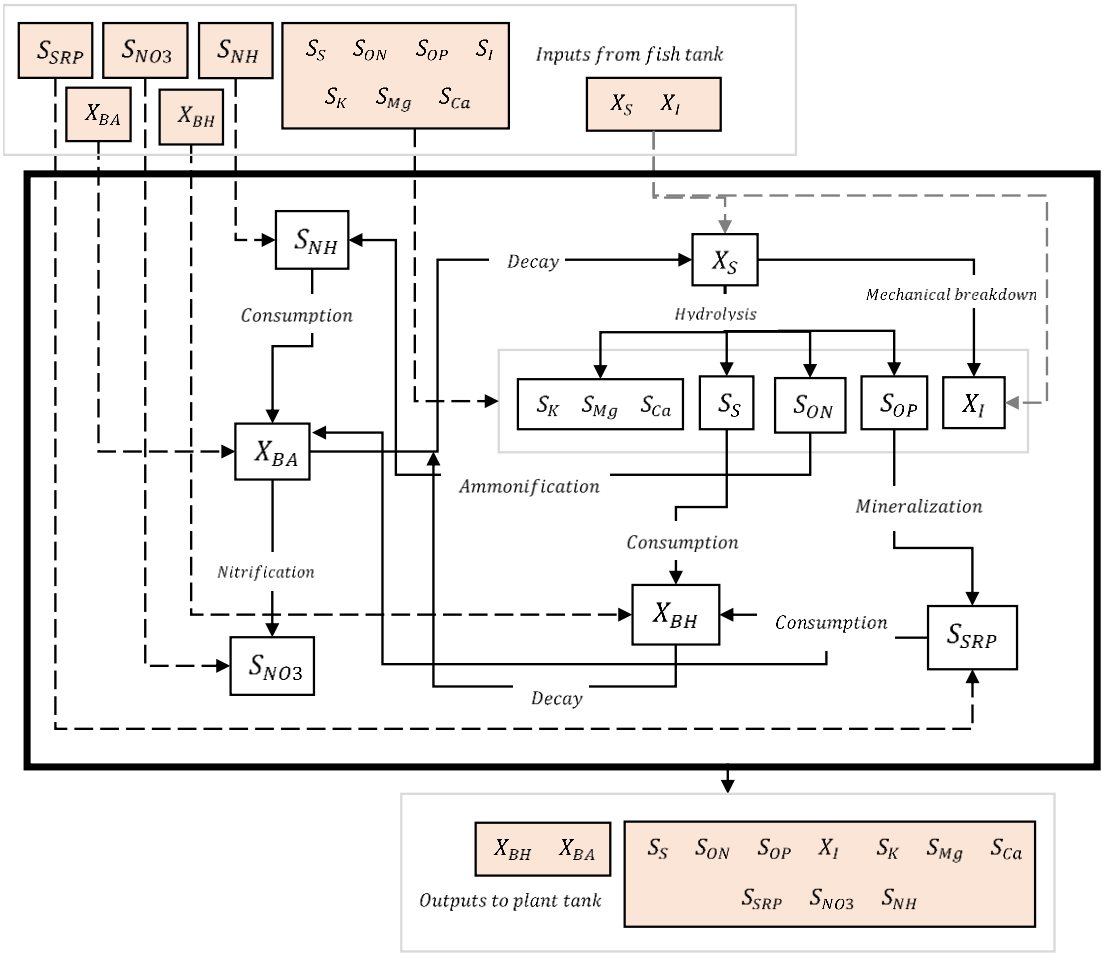

Figure 5. Bioreactor process schematic diagram

In the bioreactor, soluble organic nitrogen and phosphorus are ammonified and mineralized to ammonia and soluble reactive phosphorus, respectively, by the heterotrophic bacteria in the bioreactor. Ammonia, soluble reactive phosphorus, and soluble carbon concentrations are lowered as the bacteria grow using these substrates. The bioreactor in this model was not given a specific solids removal efficiency value, and functions according to the bacterial concentrations and specified kinetic parameters. There is no inert solids wasting

mechanism in this model, so inert mass tends to build up in the system. A wasting mechanism can be added in later iterations.

2.3.3 Plant tank schematic

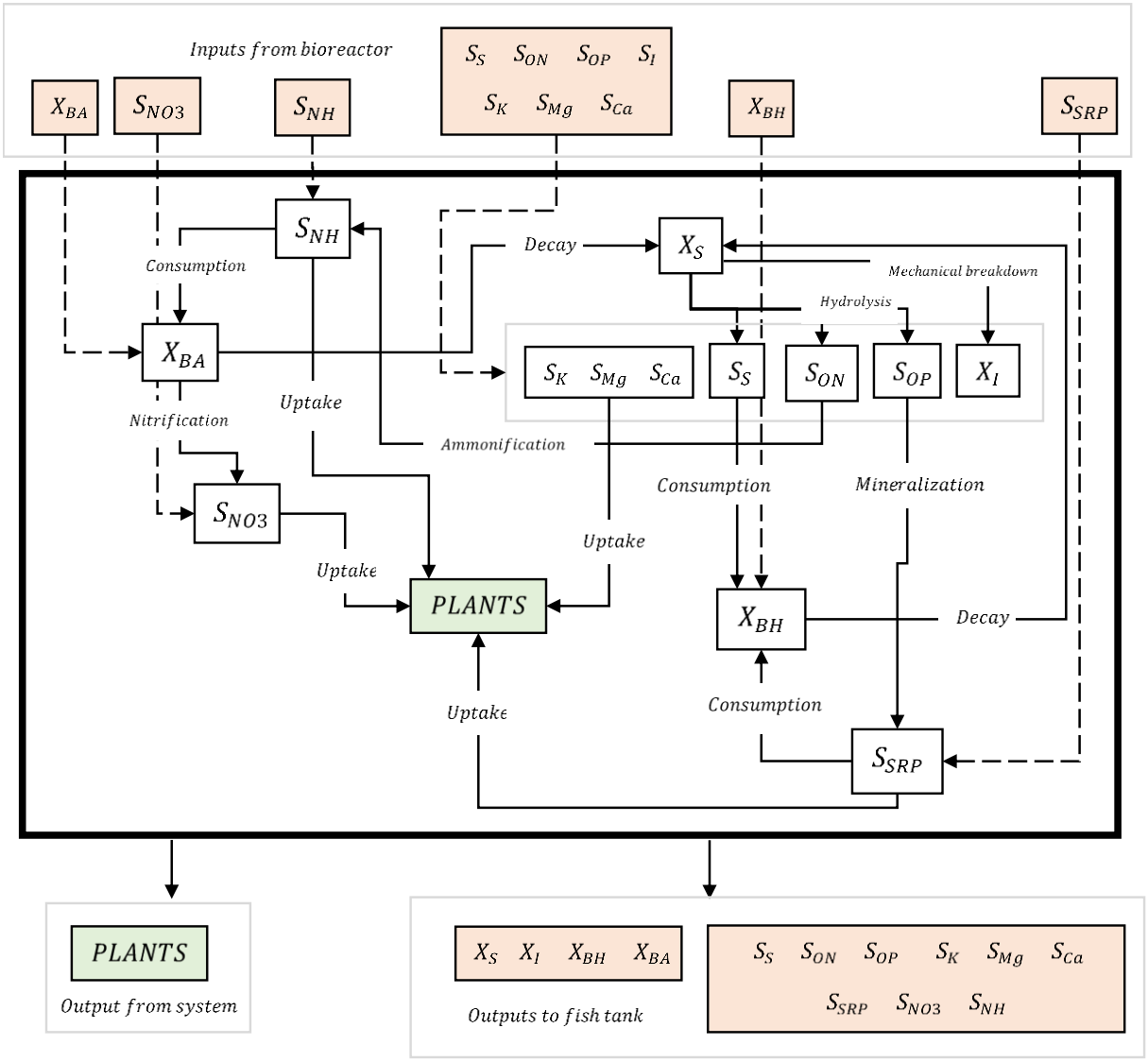

Figure 6. Schematic process diagram for the plant tank

In the plant tank, the soluble inorganic nutrients produced by fish and as the products of previous hydrolysis processes are taken up by plant roots. The same bacterial processes outlined above also take place here, as the bacteria are assumed to be allowed to flow freely between all tanks.

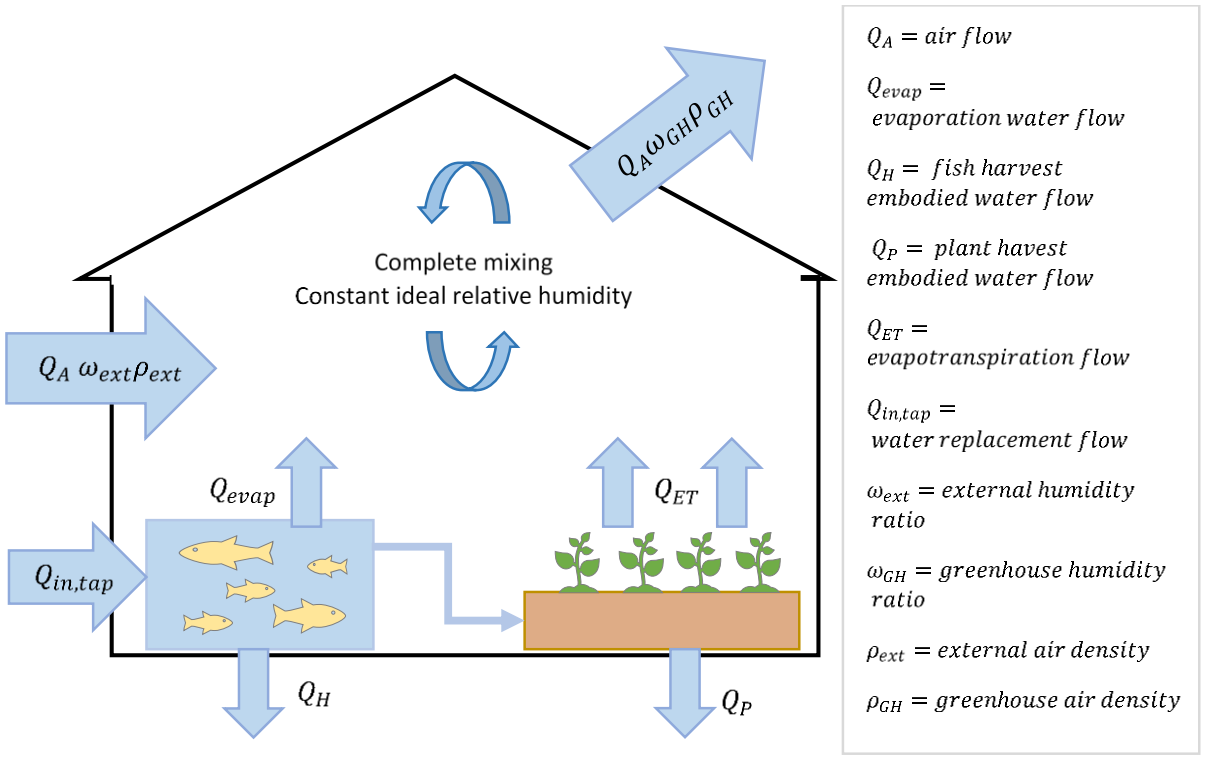

Figure 7: Water dynamic process schematic for the aquaponic CEA

Water vapor enters the CEA airspace through evaporation and evapotranspiration, and leaves the CEA as harvested plant and fish biomass, and as vapor through the roof ridge vent. The internal environment of the greenhouse is kept at a constant relative humidity idealized for plant growth, and vapor entering the greenhouse environment is assumed to be well-mixed before exiting through the roof. This results in the following conservation relationship:

Water vapor conservation (g H_2O $(day)^{-1}$)

$$\frac{dWater}{dt} = 0 = Q_A\omega_{ext}\rho_{ext} - Q_A\omega_{GH}\rho_{GH} + Q_{evap}(10^6) + Q_{ET}(10^6) \quad (2.1)$$

Where all variables are as defined in Figure 6 with values in Table 1 and Table 8. The total required airflow Q_A in and out of the system which must be achieved to maintain ideal humidity, an important variable in determining total energy requirements, is determined by:

Greenhouse air flow requirement (m^3 air $(day)^{-1}$)

$$\frac{dQ_A}{dt} = \frac{\left(-Q_{ET}(10^6) - Q_{evap}(10^6)\right)}{\omega_{ext}\rho_{ext} - \rho_{GH}\omega_{GH}} \quad (2.2)$$

CHAPTER 3. MODEL ARCHITECTURE AND SYSTEM DESIGN

3.1 Assumptions and constraints

In addition to the process assumptions already described, others regarding the construction of the model itself are important to understanding the results. The same assumptions made in the construction of the ASM models were applied here, with the addition of others for the fish and plant tanks.

All incoming fish food is assumed to be eaten by fish, with none wasted. Soluble organic phosphorus and soluble organic nitrogen are inaccessible to plants. Completely-mixed flow reactors were used to model the fish, plant, and bioreactor tanks. One large representative tank was modeled per functional unit of the system. The system water volume is constant at any given time. No reactions or biological processes take place in the piping between tanks, and transport time between them is assumed to be instantaneous. No fish or plant mortality, disease, insect damage, or predation is accounted for in this iteration of the model. The relative humidity within the greenhouse is assumed constant and ideal for plant health. Water and air temperatures are constant and ideal for all organisms. The greenhouse air is assumed to be completely mixed. Light is assumed to be provided at a constant intensity, 24 hours a day, and the spectrum at night (provided by LEDs) is assumed to be functionally identical to natural sunlight. Many of the parameters used to test the model are based on other assumptions that were made on a case-by-case basis depending on the literature availability and published data.

3.2 Tank flow balance

To construct a working model of the dynamic transformation of nutrients within the CEA, an overall flow balance of the three tanks in series was constructed (see Figure 8). Water flows from the fish tank to the bioreactor, then through the plant raft tank and back to the fish tank. A sump pump was not modeled here, so it is assumed to be functionally inert. Each sector is a completely-mixed flow reactor (CMFR).

All flows are dynamic, but the total water volume in the system stays constant and is constant in each tank. All losses in each tank are assumed to be added instantaneously via a purified tap water input. No significant water is assumed to be lost from the bioreactor. The flow volumes in and out of each tank type can therefore be calculated as follows:

Fish tank effluent (m^3 $(day)^{-1}$)

$$Q_f = Q_o - Q_{evap} - Q_H + Q_{in,tap,f} = Q_o \tag{3.1}$$

Bioreactor effluent (m^3 $(day)^{-1}$)

$$Q_b = Q_o \tag{3.2}$$

Plant raft effluent (m^3 $(day)^{-1}$)

$$Q_p = Q_o - Q_{ET} - Q_P + Q_{in,tap,p} = Q_o \tag{3.3}$$

Where Q_f, Q_b, and Q_p are the input-output flow rates through each reactor. The total required water input from a pipe therefore is:

Tap water required input (m^3 $(day)^{-1}$)

$$Q_{in,tap} = Q_{evap} + Q_{ET} + Q_H + Q_P = Q_{in,tap,f} + Q_{in,tap,p} \quad (3.4)$$

Breaking down the flows in this way allows specific water loss processes to be identified as major or minor drivers of overall water demand.

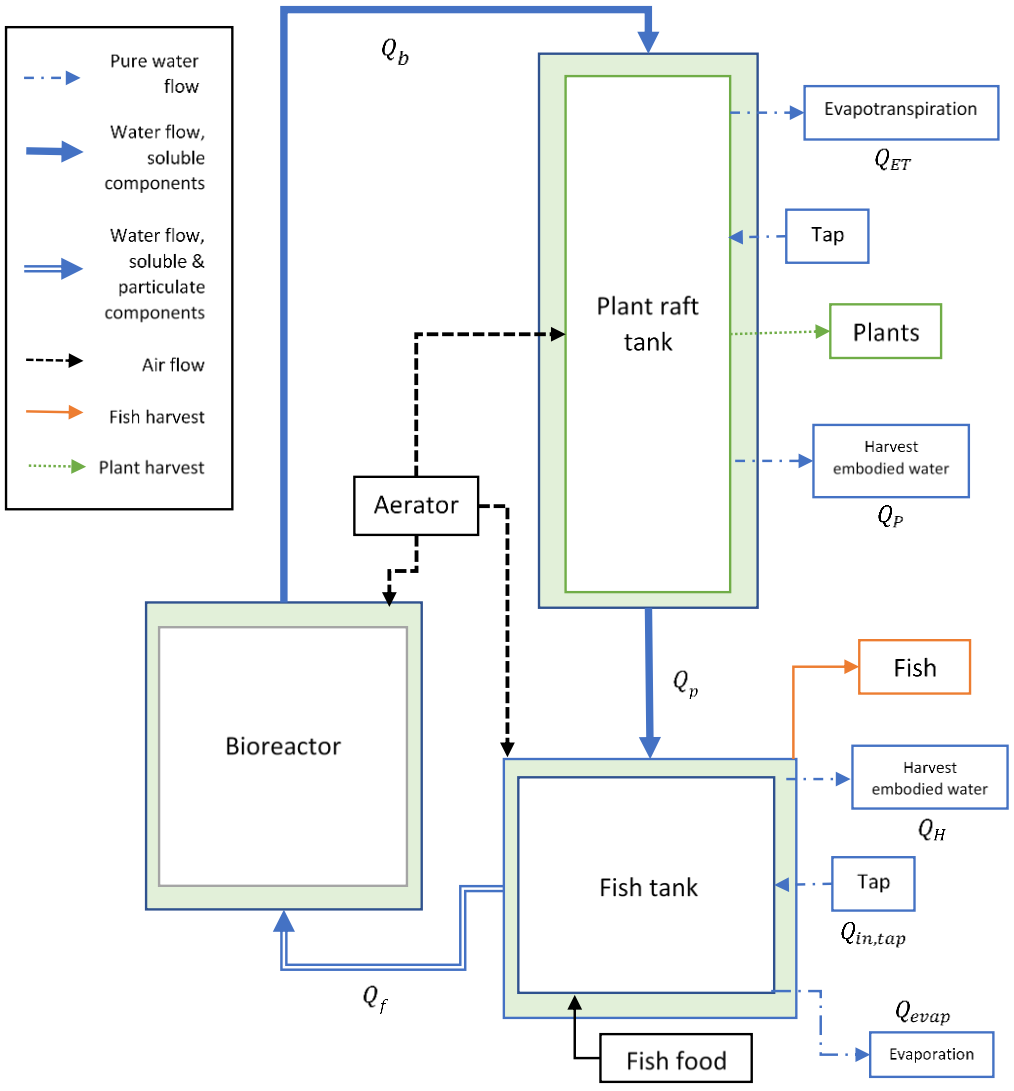

Figure 8. Tank flow schematic

CHAPTER 4. MODEL EQUATIONS

4.1 Model components

4.1.1 State variables

Table 1: System state variables

Variable	*Description*	*Units*
X_{BH}	Active heterotrophic biomass	g m^{-3}
X_{BA}	Active autotrophic biomass	g m^{-3}
S_S	Soluble BOD (easily degradable substrate)	g m^{-3}
X_S	Particulate BOD (slowly degradable substrate)	g m^{-3}
S_{ON}	Soluble organic nitrogen	g m^{-3}
S_{NH}	Ammonia-nitrogen	g m^{-3}
S_{NO3}	Nitrate	g m^{-3}
S_{OP}	Soluble organic phosphorus	g m^{-3}
S_{SRP}	Soluble reactive phosphorus	g m^{-3}
S_{O2}	Dissolved oxygen	g m^{-3}
$PlantDensity$	Plant biomass per area	g m^{-2}
C_{fish}	Fish biomass	g m^{-3}
S_K	Soluble potassium ion	g m^{-3}
S_{Ca}	Soluble calcium ion	g m^{-3}
S_{Mg}	Soluble magnesium ion	g m^{-3}
X_I	Particulate inert mass	g m^{-3}
Q_H	Water flux embodied in fish harvest	m^3 day^{-1}
Q_P	Water flux embodied in plant harvest	m^3 day^{-1}
Q_A	Air flow through greenhouse vents	m^3 $(day)^{-1}$
Q_{ET}	Water flux due to evapotranspiration	m^3 $(day)^{-1}$

4.1.2 Model parameters

Monod half-saturation parameters, fractional mass composition values, location-specific climactic parameters, and additional other values were required in order to run the

model. As many as was practical were specified here. It should be noted that the values listed here are representative or benchmark values, and care should be taken to specify or derive them for each specific system to achieve a higher degree of accuracy in predicting results. The values listed here are based on typical activated sludge system values, and literature values for lettuce (*Lactuca sativa*) and Nile tilapia *(Oreochromis nilocticus*) wherever possible. Depending on literature availability, a few were based on similar but not identical species of plant or fish, such as spinach or catfish.

Table 2: Heterotrophic parameters

Parameter	*Value*	*Definition*	*Units*	*Source*
Y_H	0.77	Yield coefficient	g X_{BH} (g S_S)$^{-1}$	[38]
k_H	2.766	Maximum substrate use coefficient	g S_S (g X_{BH}- day)$^{-}_{1}$	n/a
μ_H	2.13	Maximum specific growth rate	Day^{-1}	[38]
b_H	0.62	Decay rate	Day^{-1}	[20]
$K_{S_S,het}$	5	Half saturation coefficient for S_S dependence	g S_S m^{-3}	n/a
$K_{S_{O_2},het}$	0.2	Half saturation coefficient for oxygen dependence	g O_2 m^{-3}	[20]
$K_{NH,het}$	0.05	Half saturation coefficient for ammonia dependence	g S_{NH} m^{-3}	[20]
$K_{SRP,het}$	0.01	Half saturation coefficient for SRP dependence	g S_{SRP} m^{-3}	[20]

Most heterotrophic values were taken from typical activated sludge values, except k_H which was calculated as μ_H/Y_H. To prevent inhibition of fish growth kinetics due to high concentrations of dissolved organic carbon, $K_{S_S,het}$ had to be lowered artificially from 20 g S_S(m^{-3}) to 5 for the purposes of viewing kinetic trends over significant time periods. This parameter should be determined experimentally for an aquaponic system as it may differ from the activated sludge value.

Table 3: Autotrophic parameters

Parameter	*Value*	*Description*	*Units*	*Source*
Y_A	0.24	Yield coefficient	g S_{BA} formed g S_{NH} used^{-1}	20
μ_A	1	Maximum specific growth rate	Day^{-1}	20
k_A	4.166	Maximum substrate use coefficient	g S_{NH} used (g S_{BA} formed day)$^{-1}$	n/a
b_A	0.15	Decay rate	Day^{-1}	20
$K_{NH3,A}$	1	Half saturation coefficient for ammonia dependence	g S_{NH} m^{-3}	20
$K_{S_{O_2},A}$	0.5	Half saturation coefficient for oxygen dependence	g O_2 m^{-3}	20
$K_{SRP,A}$	0.01	Half saturation coefficient for SRP dependence	g S_{SRP} m^{-3}	20

All autotrophic parameters were based on values for nitrifying bacteria, except for k_A which was calculated as μ_A/Y_A.

Table 4: Transformation and hydrolysis rate coefficients

Parameter	*Value*	*Description*	*Units*	*Source*
k_a	0.08	Ammonification rate	(day^{-1})	39
k_m	0.22	Phosphorus mineralization rate	(day^{-1})	40
$S_{O_2,sat}$	8.2	Oxygen saturation concentration at 24°C	g O_2 (m^{-3})	41
$K_{X_S,hyd}$	0.001	Half saturation coefficient for hydrolysis of X_S	g X_S (g X_{BH})$^{-1}$	n/a
k_{hyd}	5	Maximum specific hydrolysis rate for X_S	g X_S (g X_{BH} day)$^{-1}$	n/a
$K_{S_{O_2},hyd}$	0.2	Half saturation coefficient for oxygen dependence in hydrolysis of X_S	g O_2 (m^3)	20

The ammonification rate k_a, which describes the process of transforming soluble organic nitrogen into ammonia, is a little-researched parameter[20]. Mineralization of phosphorus was estimated from a value given for the rate which occurs in natural waters[40]. The oxygen mass transfer rate coefficient $k_L a$ listed here is an inflation of usual values in order to remove any oxygen limitation to bacterial, fish, or plant growth. Several of the hydrolysis

parameters had to be altered from their ASM values to function properly in this iteration of the code, and should be determined specifically for aquaponics systems in later experiments. Hydrolysis rate was inflated for trend demonstrative purposes.

Table 5: Fractional composition parameters

Parameter	*Value*	*Description*	*Units*	*Source*
$i_{N,bio}$	0.117	Fraction of N in bacterial biomass (X_{BH} and X_{BA})	g N (g total bacterial mass)$^{-1}$	42
$i_{P,bio}$	0.052	Fraction of P in bacterial biomass (X_{BH} and X_{BA})	g P (g total bacterial mass)$^{-1}$	42
$i_{C,bio}$	0.53	Fraction of C in bacterial biomass (X_{BH} and X_{BA})	g C (g total bacterial mass)$^{-1}$	n/a
f_d	0.8	Net biodegradable fraction of biomass	g X_S (g bacterial mass)$^{-1}$	n/a
$i_{N,feces}$	0.0295	Fraction of N in feces	g N (g dry feces)$^{-1}$	43
$i_{P,feces}$	0.022	Fraction of P in feces	g P (g dry feces)$^{-1}$	43
$i_{K,feces}$	0.001	Fraction of K in feces	g K (g dry feces)$^{-1}$	44
$i_{Ca,feces}$	0.0699	Fraction of Ca in feces	g Ca (g dry feces)$^{-1}$	44
$i_{Mg,feces}$	0.0053	Fraction of Mg in feces	g Mg (g dry feces)$^{-1}$	44
$i_{C,feces}$	0.4	Fraction of C in feces	g C (g dry feces)$^{-1}$	n/a
$i_{N,Xs}$	0.0733	Fraction of N in X_S	g N g X_S^{-1}	n/a
$i_{P,Xs}$	0.037	Fraction of P in X_S	g P g X_S^{-1}	n/a
$i_{C,Xs}$	0.465	Fraction of C in Xs	g C g Xs^{-1}	n/a
$i_{N,food}$	0.063	Fraction of N in fish food	g N (g food)$^{-1}$	45
$i_{P,food}$	0.022	Fraction of P in fish food	g P (g food)$^{-1}$	45
$i_{C,food}$	0.4	Fraction of C in fish food	g C (g food)$^{-1}$	45
$i_{K,food}$	0.012	Fraction of K^+ in fish food	g K (g food)$^{-1}$	45
$i_{Mg,food}$	0.00354	Fraction of Mg^{2+} in fish food	g Mg (g food)$^{-1}$	45
$i_{Ca,food}$	0.041	Fraction of Ca^{2+} in fish food	g Ca (g food)$^{-1}$	45
$i_{water,fish}$	0.8	Fraction of water in fish biomass	g water (g fish fresh weight)$^{-1}$	46
$i_{N,fish}$	0.0979	Fraction of N in fish tissue	g N (g dry fish)$^{-1}$	47
$i_{N,plant}$	0.0584	Fraction of N in plant biomass	g N (g dry plant)$^{-1}$	45
$i_{P,plant}$	0.0104	Fraction of P in plant biomass	g P (g dry plant)$^{-1}$	45
$i_{K,plant}$	0.0390	Fraction of K in plant biomass	g K (g dry plant)$^{-1}$	45

Table 5 continued

$i_{Mg,plant}$	0.0036	Fraction of Mg in plant biomass	g Mg (g dry plant)$^{-1}$	45
$i_{Ca,plant}$	0.0122	Fraction of Ca in plant biomass	g Ca (g dry plant)$^{-1}$	45
$i_{water,plant}$	0.95	Fraction of water in plant biomass	g water (g fresh weight)$^{-1}$	n/a

The fractions of N, P, and C in X_S were all calculated as the sum of fractions in bacterial biomass and feces, since these two components make up the total organic substrate mass in the system. The fraction of net biodegradable material in bacterial biomass and fractions of water in fish and plant tissue were guessed. Fractions of each nutrient in fish food were based on studies of commercial fish feed but had to be altered to facilitate idealized fish growth rates and a suitably high N-loading rate to the system.

Table 6: Plant parameters

Parameter	*Value*	*Description*	*Units*	*Source*
Y_{plant}	0.1992	Yield coefficient	g plant (g nutrients added^{-1}	estimate based on 48
k_{plant}	6.526	Maximum substrate use rate	g nutrients (g plant – day)$^{-1}$	n/a
μ_{plant}	1.3	Maximum specific growth rate	Day^{-1}	49
b_{plant}	0.0265	Respiration/decay rate	Day^{-1}	49
$K_{PPFD,plant}$	200	Half saturation coefficient for light intensity dependence	µmol photons (m^2 s)$^{-1}$	35
$K_{I,PPFD,plant}$	1600	Haldane inhibition coefficient for light intensity dependence	µmol photons (m^2 s)$^{-1}$	estimate based on 30, 35
$K_{NH3,plant}$	10	Half saturation coefficient for ammonia dependence	g NH3 m^{-3}	n/a
$K_{NO3,plant}$	4	Half saturation coefficient for nitrate dependence	g NO3$^-$ m^{-3}	n/a
$K_{SRP,plant}$	3	Half saturation coefficient for SRP dependence	g SRP m^{-3}	~~n/a~~
$K_{K,plant}$	1	Half saturation coefficient for potassium dependence	g K m^{-3}	n/a

Table 6 continued

$K_{Ca,plant}$	1	Half saturation coefficient for calcium dependence	g Ca m^{-3}	n/a
$K_{Mg,plant}$	0.243	Half saturation coefficient for magnesium dependence	g Mg m^{-3}	[50]
$K_{SO_2,plant}$	5	Half saturation coefficient for dissolved oxygen dependence	g S_{O2} m^{-3}	[30]
$K_{PCO_2,plant}$	0.0003	Half saturation coefficient for carbon dioxide dependence	atm	[35]

Unlike bacteria, plants grow at dynamic rates which cannot be described by a single constant yield coefficient. The Y_{plant} listed here is a benchmark value used until a more representative parameter can be determined, and is the result of a calculation relating final dry plant mass (harvest size) to the total mass of all added fertilizers. In future iterations of the model, plants of different sizes with different yield coefficients could be treated in separate groups to make this modeling method more realistic. The maximum substrate use rate was calculated as μ_{plant}/Y_{plant}. Benchmark values found in literature for the plant growth parameters were extrapolated using data from a variety of different sources with varying degrees of applicability to the current model. The parameters were adjusted here according to their expected behavior in favor of maximizing plant growth, and many had to be estimated based on concentrations of soluble nutrients occurring in the system. Future CEA-specific studies could more accurately determine plant sensitivity to each soluble nutrient in an aquaponics setting.

Table 7: Fish parameters

Parameter	*Value*	*Description*	*UNITS*	*Source*
Y_{fish}	0.6952	Yield coefficient	g fish (g food)$^{-1}$	[51, 52]
μ_{fish}	0.0139	Maximum specific growth rate	Day^{-1}	n/a

Table 7 continued

b_{fish}	0.00149	Fasting catabolism rate	$g^{1-n}(day)^{-1}$	29
k_{fish}	0.02	Maximum specific substrate use rate of fish	g food (g fish-day)$^{-1}$	n/a
$K_{F,N}$	0.01	Half saturation coefficient for N in food dependence	g N (g food)$^{-1}$	53,51,54,55
$K_{F,P}$	0.00233	Half saturation coefficient for P in food dependence	g P (g food)$^{-1}$	56
$K_{F,C}$	0.1	Half saturation coefficient for C in food dependence	g C (g food)$^{-1}$	54,55,57
$K_{F,K}$	0.009493	Half saturation coefficient for K^+ in food dependence	g K^+ (g food)$^{-1}$	58
$K_{F,Ca}$	0.005	Half saturation coefficient for Ca^{2+} in food dependence	g Ca^{2+} (g food)$^{-1}$	n/a
$K_{F,Mg}$	0.0003715	Half saturation coefficient for Mg^{2+} in food dependence	g Mg^{2+} (g food)$^{-1}$	59
$K_{S_S,fish}$	30	Half saturation coefficient for Ss dependence	g S_S m^{-3}	28
$K_{SO_2,fish}$	0.898	Half saturation coefficient for dissolved oxygen dependence	g O_2 m^{-3}	28,29

To calculate b_{fish}, an expression was used from literature[29] which describes the dependence of this 'fasting catabolism' rate on water temperature: $b_{fish} = 0.00133e^{0.0132(T-15)}$.

Table 8: Physical and environmental parameters

Variable	*Value*	*Description*	*Units*	*Source*
A_{fish}	13.378	Sum of areas of all fish tanks	m^2	GT testbed
d_{fish}	1.2192	Depth of fish tanks	m	GT testbed
V_{fish}	16.31	Volume of fish tanks	m^3	GT testbed
$A_{bioreactor}$	16.31	Area of bioreactor	m^2	GT testbed
$d_{bioreactor}$	0.5	Depth of bioreactor	m	GT testbed
$V_{bioreactor}$	8.155	Volume of bioreactor	m^3	GT testbed

Table 8 continued

A_{plant}	147.158	Sum of areas of all plant rafts	m^2	GT testbed
d_{plant}	0.3048	Depth of the plant rafts	m	GT testbed
V_{plant}	44.85	Volume of plant rafts	m^3	GT testbed
T_w	*24*	Water temperature	°C	n/a
T_{GH}	24	Greenhouse air temperature	°C	n/a
T_{max}	30	Maximum daily temperature	°C	n/a
T_{min}	20	Minimum daily temperature	°C	n/a
T_{ext}	19	Temperature outside the greenhouse	°C	n/a
P_{atm}	101.325	Atmospheric pressure	kPa	n/a
DEG	33.749	Latitude	°	[60]
$PPFD$	1500	Photosynthetic photon flux density	µmol photons $(m^2\ s)^{-1}$	n/a
P_{CO_2}	$10^{-3.42}$	Atmospheric carbon dioxide	atm	n/a
ω_{ext}	0.01	Humidity ratio of the outdoor air	g H_2O (g dry air)$^{-1}$	[61]
ω_{GH}	0.014	Humidity ratio of air in greenhouse (well-mixed)	g H_2O (g dry air)$^{-1}$	[61]
ρ_{ext}	1.1682	Density of external air	g $(m^3)^{-1}$	[37]
ρ_{GH}	1.1492	Density of greenhouse air	g $(m^3)^{-1}$	[37]
$k_L a$	22	Oxygen mass transfer rate into water for submerged air stone	Day^{-1}	n/a
Q_{evap}	0.1565	Water flow due to evaporation	$m^3\ day^{-1}$	[36]
Q_o	100	Water flow between all three tanks	$m^3\ day^{-1}$	n/a
Δ	0.1790	Slope of saturation vapor pressure curve at T_{GH}	kPa $(°C^{-1})$	[37]
R_n	≈ 11	Net radiation	MJ $(m^2$-day)	[37]
G	0	Soil heat flux	MJ $(m^2$-day)	[37]
ρ_a	0.0011	Mean air density at T_{GH} and P_{atm}	kg $(m^3)^{-1}$	[37]
c_p	0.001	Specific heat of air	MJ $(m^2$-°C$)^{-1}$	[37]
e_s	2.9839	Saturation vapor pressure at T_{GH}	kPa	[37]

e_a	2.0887	Actual vapor pressure at T_{GH}	kPa	[37]
r_a	13.046	Aerodynamic resistance	s m^{-1}	[37]
r_s	70	Bulk surface resistance	s m^{-1}	[37]
γ	0.0674	Psychrometric constant	kPa $(°C)^{-1}$	[37]
λ	2.45	Latent heat of vaporization of water	MJ kg^{-1}	[37]

4.2 Rate equations

The rate equations listed in this section govern each individual process taking place in the system. They are combined into a series of differential equations, which form the basis of the model.

Net active heterotrophic biomass growth (g biomass $(m^3\text{-day})^{-1}$)

$$r_{het} = k_{het} Y_{het} \left[\frac{S_S}{K_{S_S,het} + S_S}\right]\left[\frac{S_{O_2}}{K_{S_{O_2},het} + S_{O_2}}\right]\left[\frac{S_{NH}}{K_{S_{NH},het} + S_{NH}}\right]\left[\frac{S_{SRP}}{K_{S_{SRP},het} + S_{SRP}}\right] X_{BH} \quad (4.1)$$

Viable net autotrophic biomass growth (g biomass $(m^3\text{-day})^{-1}$)

$$r_A = k_A Y_A \left[\frac{S_{O_2}}{K_{S_{O_2,A}} + S_{O_2}}\right]\left[\frac{S_{NH}}{K_{S_{NH},A} + S_{NH}}\right]\left[\frac{S_{SRP}}{K_{S_{SRP},A} + S_{SRP}}\right] X_{BA} \quad (4.2)$$

Heterotrophic biomass decay (g biomass $(m^3\text{-day})^{-1}$)

$$r_{het\ decay} = b_H X_{BH} \quad (4.3)$$

Autotrophic biomass decay (g biomass $(m^3\text{-day})^{-1}$)

$$r_{A\ decay} = b_A X_{BA} \tag{4.4}$$

Heterotrophic carbon respiration (g C $(m^3\text{-day})^{-1}$)

$$r_{hetresp} = r_{het} X_{BH} (1.947) \left(\frac{12}{44}\right) \tag{4.5}$$

Hydrolysis of X_S into S_S (g X_S $(m^3\text{-day})^{-1}$)

$$r_{hyd_{X_S}} = k_{hyd} \left[\frac{S_{O_2}}{K_{H,S_{O_2},} + S_{O_2}}\right] \frac{\frac{X_S}{X_{BH}}}{K_{X_S,hyd} + \frac{X_S}{X_{BH}}} X_{BH} \tag{4.6}$$

Hydrolysis of X_S into S_{ON} (X_{ON} $(m^3\text{-day})^{-1}$)

$$r_{hyd_N} = r_{hyd_{X_S}} i_{N,X_s} \tag{4.7}$$

Hydrolysis of X_S into S_{OP} (X_{OP} $(m^3\text{-day})^{-1}$)

$$r_{hyd_P} = r_{hyd_{X_S}} i_{P,X_S} \tag{4.8}$$

Mineralization of S_{OP} into S_{SRP} (g S_{OP} $(m^3\text{-day})^{-1}$)

$$r_{mineral} = k_m (S_{OP}) \tag{4.9}$$

Ammonification of S_{ON} into S_{NH} (g S_{NH} $(m^3\text{-day}^{-1})$)

$$r_{ammonification} = k_a (S_{ON}) X_S \tag{4.10}$$

Fish feed rate (g food $(m^3\text{-day})^{-1}$)

$$r_{food} = 0.02 C_{fish} \tag{4.11}$$

Fish growth (whole fish) (g fish dry mass (m^3-day)$^{-1}$)

$$r_{fish} = k_{fish} Y_{fish} \left[\frac{\boldsymbol{i_{C,food}}}{K_{F_C} + \boldsymbol{i_{C,food}}}\right]\left[\frac{i_{N,food}}{K_{F_N} + i_{N,food}}\right]\left[\frac{i_{P,food}}{K_{F_P} + i_{P,food}}\right]\left[\frac{S_{O_2}}{K_{S_{O_2},fish} + S_{O_2}}\right] \left[\frac{K_{S_S,fish}}{K_{S_S,fish} + S_S}\right]\left[\frac{i_{K,food}}{K_{F_K} + i_{K,food}}\right]\left[\frac{i_{Ca,food}}{K_{F_{Ca}} + i_{Ca,food}}\right]\left[\frac{i_{Mg,food}}{K_{F_{Mg}} + i_{Mg,food}}\right] C_{fish} \tag{4.12}$$

Fish respiration (g O_2 (m^3-day)

$$r_{fishresp} = r_{fish}\left(\frac{-1 - Y_{fish}}{Y_{fish}}\right) \tag{4.13}$$

Ammonia excretion via fish gills (g N (m^3-day)$^{-1}$)

$$r_{NH_{gills}} = 0.02 C_{fish} V_{fish} (0.8) \tag{4.14}$$

Fish catabolism (g fish (m^3-day)$^{-1}$)

$$r_{catabolism} = b_{fish}\left(C_{fish}\right)^{0.81} \tag{4.15}$$

Fish feces carbon excretion (g C (m^3-day)$^{-1}$)

$$r_{feces_C} = r_{food} i_{C,food} - r_{fish} i_{C,fish} - r_{fishresp} - r_{catabolism} i_{C,fish} \tag{4.16}$$

Fish feces nitrogen excretion (g N (m^3-day)$^{-1}$)

$$r_{feces_N} = r_{food} i_{N,food} - r_{fish} i_{N,fish} - r_{NH,gills} - r_{catabolism} i_{N,fish} \tag{4.17}$$

Fish feces phosphorus excretion (g P $(m^3\text{-day})^{-1}$)

$$r_{feces_P} = r_{food} i_{P,food} - r_{fish} i_{P,fish} \tag{4.18}$$

Fish feces potassium excretion (g K $(m^3\text{-day})^{-1}$)

$$r_{feces_K} = r_{food} i_{K,food} - r_{fish} i_{K,fish} \tag{4.19}$$

Fish feces calcium excretion (g Ca $(m^3\text{-day})^{-1}$)

$$r_{feces_{Ca}} = r_{food} i_{Ca,food} - r_{fish} i_{Ca,fish} \tag{4.20}$$

Fish feces magnesium excretion (g Mg $(m^3\text{-day})^{-1}$)

$$r_{feces_{Mg}} = r_{food} i_{Mg,food} - r_{fish} i_{Mg,fish} \tag{4.21}$$

Fish feces inert mass excretion (g inert mass $(m^3\text{-day})^{-1}$)

$$r_{fecesinert} = 0.1914 r_{food} \tag{4.22}$$

Fish harvest (g fish $(m^3\text{-day})^{-1}$) (equal to net growth rate)

$$r_{fish_{harvest}} = r_{fish} C_{fish} - r_{catabolism} \tag{4.23}$$

Plant growth (phenotype + roots) (g plant dry mass (m^2-day)$^{-1}$)

$$r_{plant} = k_{plant} Y_{plant} \left[\frac{PPFD}{K_{PPFD}+PPFD+\frac{PPFD^2}{K_{I,PPFD}}}\right]\left[\frac{S_{O_2}}{K_{S_{O_2},plant}+S_{O_2}}\right]$$

$$\left[\frac{P_{CO_2}}{K_{P_{CO_2},plant}+P_{CO_2}}\right]\left[\frac{S_{NH_3}}{K_{S_{NH_3},plant}+S_{NH_3}}\right]\left[\frac{S_{NO_3}}{K_{S_{NO_3}}+S_{NO_3}}\right]\left[\frac{S_{SRP}}{K_{S_{SRP},plant}+S_{SRP}}\right]\left[\frac{S_K}{K_{K\ plant}+S_K}\right] \quad (4.24)$$

$$\left[\frac{S_{Mg}}{K_{Mg\ plant}+S_{Mg}}\right]\left[\frac{S_{Ca}}{K_{Ca\ plant}+S_{Ca}}\right]\left[\frac{T_{GH}}{K_{T_{GH}}+T_{GH}+\frac{T_{GH}^2}{K_{I,T_{GH}}}}\right] PlantDensity$$

Plant respiration (g plant dry mass (m^2-day)$^{-1}$)

$$r_{plantresp} = b_{plant} PlantDensity \quad (4.25)$$

Plant harvest (phenotype + roots) (g plant (m^2-day)$^{-1}$)

$$r_{plant_{harvest}} = r_{plant} - r_{plantresp} \quad (4.26)$$

Dissolution of oxygen into water (g O_2 (m^3-day)$^{-1}$)

$$r_{aeration} = k_L a\left(S_{O_2,sat} - S_{O_2}\right) \quad (4.27)$$

Evapotranspiration rate (Penman-Monteith equation) (m^3 H_2O (day)$^{-1}$)

$$Q_{ET} = \left(\frac{\Delta(R_n - G) + \rho_a c_p \frac{(e_s - e_a)}{r_a}}{\Delta + \lambda\gamma\left(1 + \frac{r_s}{r_a}\right)}\right)\left(\frac{A_{plant}}{1000}\right) \quad (4.28)$$

Addition of water vapor via ventilation (g H_2O (day)$^{-1}$)

$$r_{vent,in} = Q_A(\omega_{ext})\rho_{ext} \tag{4.29}$$

Loss of water vapor via ventilation (g H_2O $(day)^{-1}$)

$$r_{vent,out} = Q_A(\omega_{GH})\rho_{GH} \tag{4.30}$$

Embodied water in fish harvest (g H_2O $(day)^{-1}$)

$$Q_H = r_{fish_{harvest}} i_{water,fish} C_{fish} V_{fish} \tag{4.31}$$

Embodied water in plant harvest (g H_2O $(day)^{-1}$)

$$Q_P = r_{plant_{harvest}}\, i_{water,plant}(PlantDensity) A_{plant} \tag{4.32}$$

4.2.1 Biokinetic matrices of rate equations

The IWA model uses a matrix format to describe the complex interactions of species and their stoichiometric relationships in the ASM models. Complete documentation of the reasoning for this notation and description of the matrix format can be found within these models[20], and a similar method is used here. The change in concentration of each species (along the top row) is calculated by multiplying each process term (far right column) by the stoichiometric relationship given in the intersection box of process and species, and summing these for each process acting on that species (moving down the column).

Component → Process ↓	X_S	S_S	X_{BH}	X_{BA}	S_{NH}	S_{NO_3}	S_{SRP}	S_{ON}	S_{OP}	S_{O2}	S_K	S_{Ca}	S_{Mg}	X_I	$Fish$	Process rate
Heterotrophic growth		$-\frac{1}{Y_{het}}$	1		$-i_{N,bio}$		$-i_{P,bio}$			$-\frac{(1-Y_{het})}{Y_{het}}$						r_{het}
Heterotrophic decay	1		-1													$r_{het\ decay}$
Heterotrophic respiration		-1														$r_{hetresp}$
Autotrophic growth				1	$-i_{N,bio} - \frac{1}{Y_A}$	$\frac{1}{Y_A}$	$-i_{P,bio}$			$-\frac{4.57-Y_A}{Y_A}$						r_A
Autotrophic decay	1			-1												$r_{A\ decay}$
Hydrolysis of X_S to S_S	-1	1														$r_{hyd_{X_S}}$
Hydrolysis of X_S to S_{ON}	-1							1								r_{hyd_N}
Hydrolysis of X_S to S_{OP}	-1								1							r_{hyd_P}
Mineralization of S_{OP}							1		-1							$r_{mineral}$
Ammonification of S_{ON}					1			-1								$r_{ammonification}$
S_{NH} Gill excretion					1											$r_{NH_{gills}}$
Oxygen mass transfer to water										1						$r_{aeration}$
Fish growth										$\frac{-1-Y_{fish}}{Y_{fish}}$					1	r_{fish}
Feces C excretion	1															$r_{fece\ s_C}$
Feces N excretion	1															$r_{fece\ s_N}$
Feces P excretion	1															$r_{fece\ s_P}$
Feces K excretion											1					$r_{fece\ s_K}$
Feces Ca excretion												1				$r_{fece\ s_{Ca}}$
Feces Mg excretion													1			$r_{fece\ s_{Mg}}$
Fish harvest															-1	$r_{fish_{harvest}}$
Fish catabolism															-1	$r_{catabolism}$
Fish feeding														0.1914		r_{food}

Figure 9. Biokinetic matrix for the fish tank

Component → Process ↓	X_S	S_S	X_{BH}	X_{BA}	S_{NH}	S_{NO_3}	S_{SRP}	S_{ON}	S_{OP}	S_{O2}	Process rate
Heterotrophic growth		$-\frac{1}{Y_{het}}$	1		$-i_{N,bio}$		$-i_{P,bio}$			$-\frac{(1-Y_{het})}{Y_{het}}$	r_{het}
Heterotrophic decay	1		-1								$r_{het\ decay}$
Heterotrophic respiration		-1									$r_{hetresp}$
Autotrophic growth				1	$-i_{N,bio} - \frac{1}{Y_A}$	$\frac{1}{Y_A}$	$-i_{P,bio}$			$-\frac{4.57-Y_A}{Y_A}$	r_A
Autotrophic decay	1			-1							$r_{A\ decay}$
Hydrolysis of X_S to S_S	-1	1									$r_{hyd_{X_S}}$
Hydrolysis of X_S to S_{ON}	-1							1			r_{hyd_N}
Hydrolysis of X_S to S_{OP}	-1								1		r_{hyd_P}
Mineralization of S_{OP}							1		-1		$r_{mineral}$
Ammonification of S_{ON}					1			-1			$r_{ammonification}$
Oxygen mass transfer to water										1	$r_{aeration}$

Figure 10. Biokinetic matrix for the bioreactor

Component → Process ↓	X_S	S_S	X_{BH}	X_{BA}	S_{NH}	S_{NO_3}	S_{SRP}	S_{ON}	S_{OP}	S_{O2}	S_K	S_{Ca}	S_{Mg}	Plants	Process rate
Heterotrophic growth		$-\frac{1}{Y_{het}}$	1		$-i_{N,bio}$		$-i_{P,bio}$			$-\frac{(1-Y_{het})}{Y_{het}}$					r_{het}
Heterotrophic decay	1		-1												$r_{het\ decay}$
Heterotrophic respiration		-1													$r_{hetres\ p}$
Autotrophic growth				1	$-i_{N,bio}$ $-\frac{1}{Y_A}$	$\frac{1}{Y_A}$	$-i_{P,bio}$			$-\frac{4.57-Y_A}{Y_A}$					r_A
Autotrophic decay	1			-1											$r_{A\ deca\ y}$
Hydrolysis of X_S to S_S	-1	1													$r_{hyd_{X_S}}$
Hydrolysis of X_S to S_{ON}	-1							1							r_{hyd_N}
Hydrolysis of X_S to S_{OP}	-1								1						r_{hyd_P}
Mineralization of S_{OP}							1		-1						$r_{mineral}$
Ammonification of S_{ON}					1			-1							$r_{ammonification}$
Oxygen mass transfer to water										1					$r_{aeration}$
Plant growth						$-i_{N,plant}$	$-i_{P,plant}$				$-i_{K,plant}$	$-i_{Ca,plant}$	$-i_{Mg,plant}$	1	r_{plant}
Plant respiration														-1	$r_{plantresp}$
Plant harvest														-1	$r_{plan\ t_{harvest}}$

Figure 11. Biokinetic matrix for the plant tank

CHAPTER 5. RESULTS

The model, built in MATLAB, was successful in producing results which showed the expected kinetic relationships between species and processes. The results shown here were achieved for a somewhat idealized system with high-nutrient-density fish feed, and certain modifications to parameters which would exaggerate kinetic behavior for modeling purposes. Importantly, the parameters with which the model was run are highly variable and very specific to each individual system, fish and plant species, water and air temperature, and other conditions.

5.1 Initial conditions

For the 25-day results shown here, fish density was set at 68 g/m^3 and was assumed to stay constant throughout the timespan of the model: growth rate was set equal to harvest rate. The plants were assumed not to be planted until day 10, which would allow the autotrophs time to accumulate nitrate within the system. Once added to the system, the plant density was 101 g/m^2, assumed homogeneous across all plant tanks, and constant, with growth rate equal to harvest rate. Autotrophic and heterotrophic bacteria were assumed to be initially present in the system at concentrations of 0.1 and 1 g/m^3 respectively. The full list of initial conditions, which are specific to each tank type, can be found in the solver code in Appendix A.

5.2 Model outputs

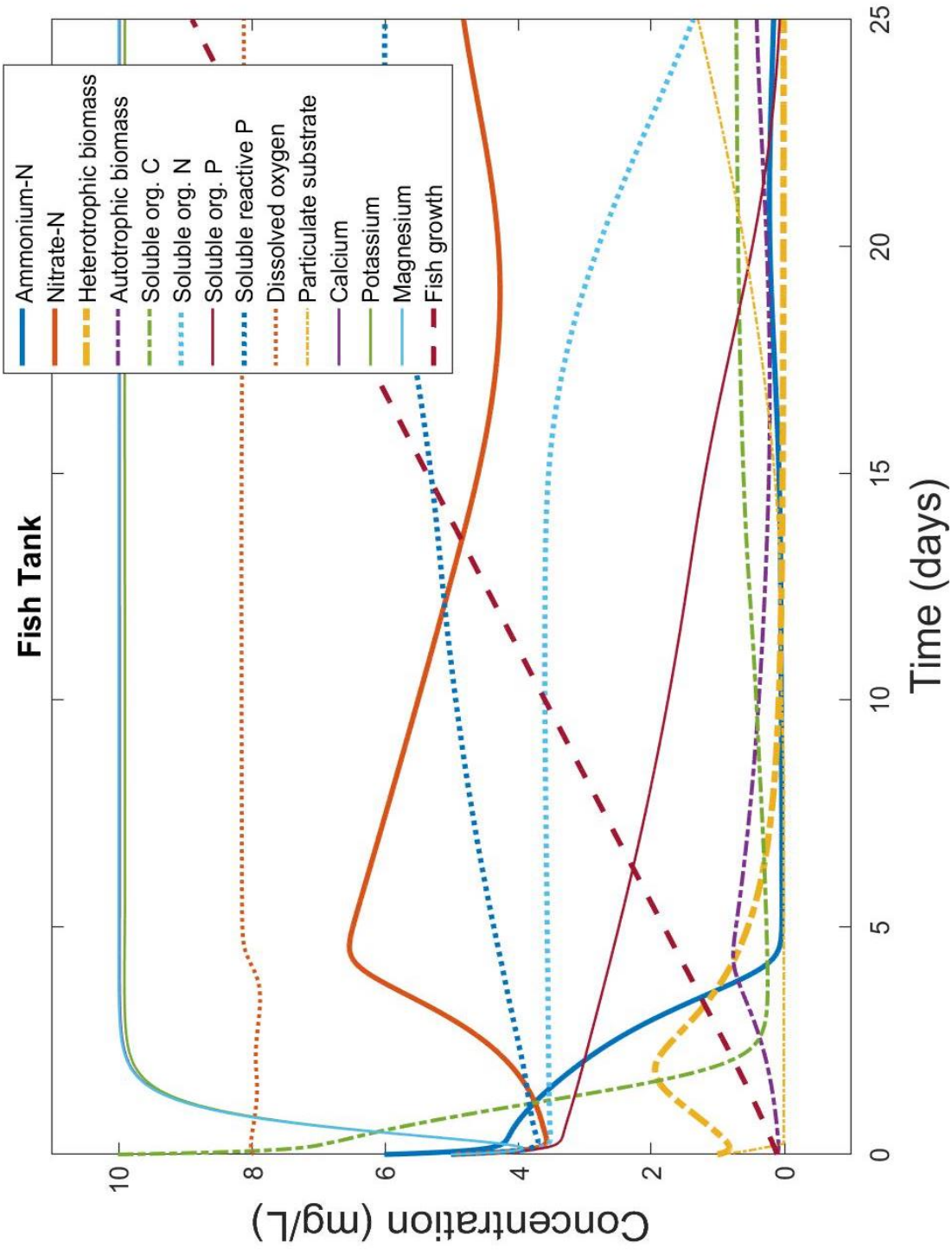

Figure 12. Fish tank output, 25 days

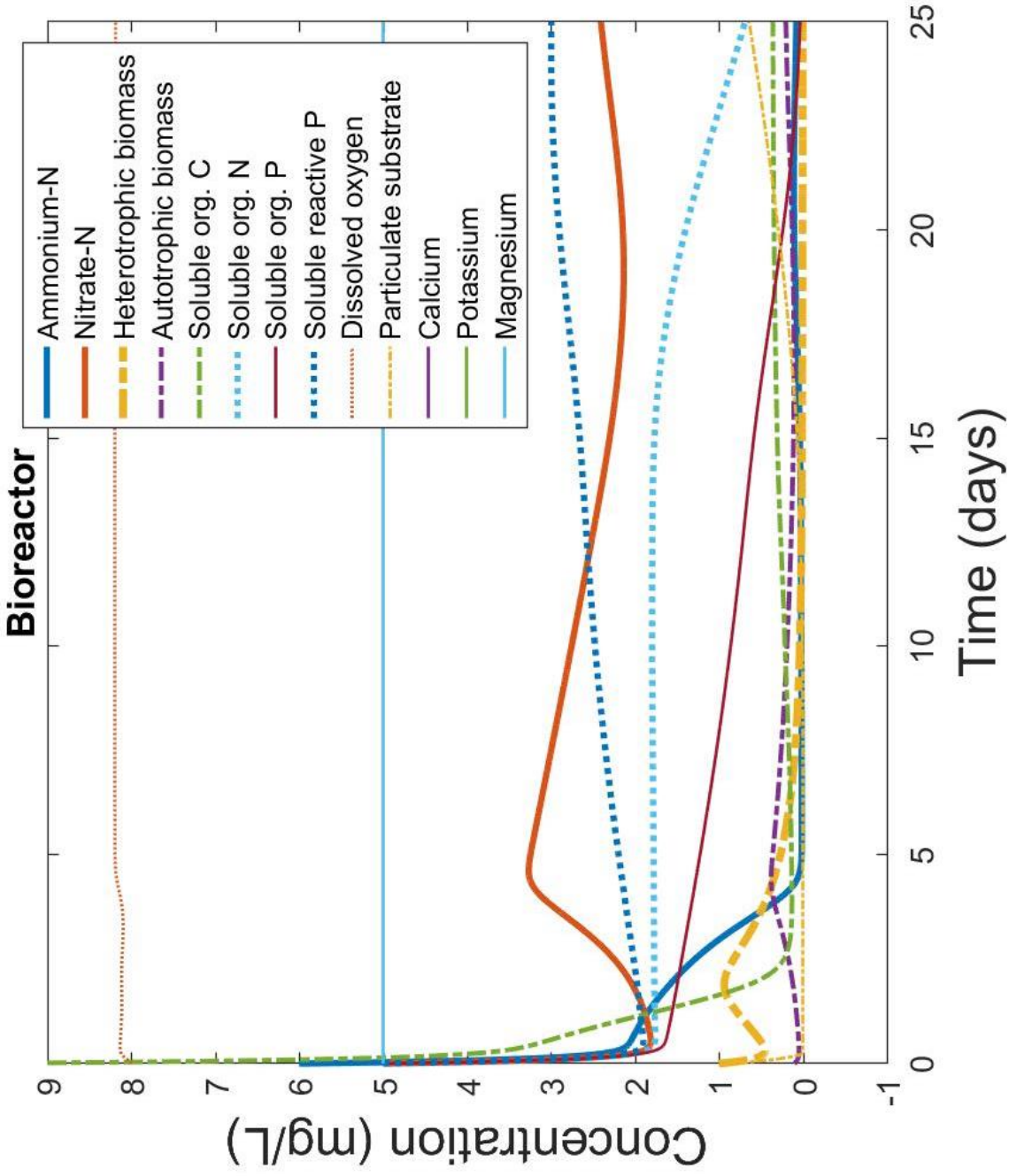

Figure 13. Bioreactor output, 25 days

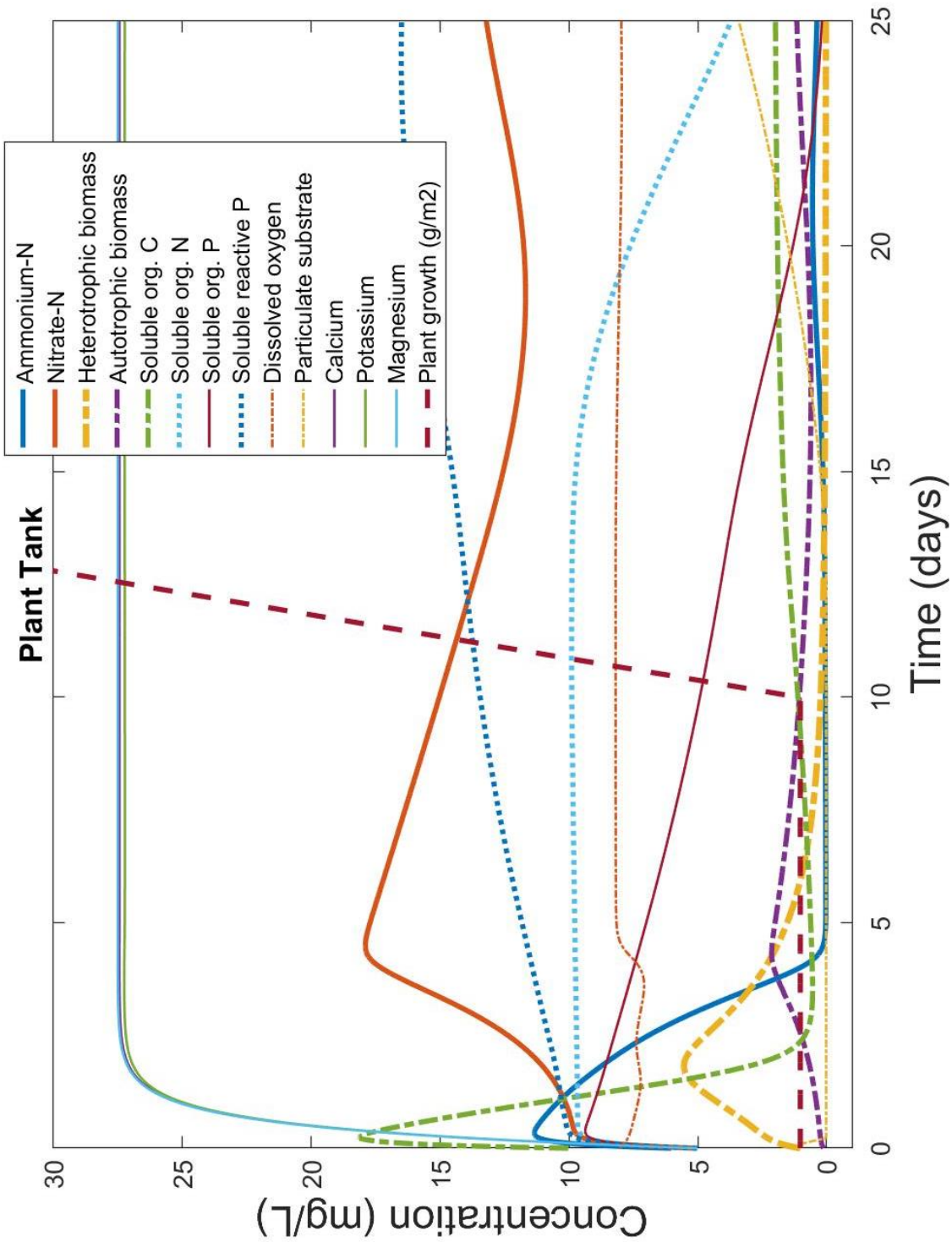

Figure 14. Plant tank output, 25 days

The results show a nearly constant growth of both fish and plants over time in an overall nitrogen-limited system. As was previously mentioned, initial conditions were optimized such that no shortage of any given food nutrient or environmental condition severely limited fish or plant growth, and fish were fed according to their maximum potential rate of consumption.

Oxygen, calcium, magnesium, and potassium all reach a high steady state concentration by the first few days. Heterotrophs respond to high initial levels of soluble carbon (S_S), peaking around 3 days, before falling and reaching a stable concentration as more carbon is slowly added via hydrolysis of fish wastes. Similarly, autotrophs deplete the ammonium nitrogen, peaking around 5 days before falling to a low steady state concentration as ammonium is slowly added via fish wastes. Particulate substrate (X_S) is held at a low level by the heterotrophs which hydrolyze it into S_S, S_{ON}, and S_{OP}. S_{ON} is ammonified into S_{NH}, which is quickly removed by the nitrifiers and heterotrophs and maintained at a low level. Subsequently, the nitrifiers produce S_{NO3}, which is then removed by plants. As a method for increasing the functionality and flexibility of the code, a conditional statement for low ammonium-N concentrations was added such that when S_{NH} concentrations are very low, the heterotrophic bacteria switch to utilizing nitrate-N as their nitrogen source rather than ammonia. S_{OP} is subsequently mineralized into S_{SRP}, which reaches a stable concentration via its removal by bacteria and plants. Inert material X_I builds up in the tank, as predicted by this iteration of the model. Its high concentration was neglected as inhibitory to the growth of any organism.

Starting around day 17, it is observed that the heterotrophic population is extremely small, causing a cascade of other species to react accordingly: X_S begins to rise due to lack

of hydrolysing heterotrophs, while the products of hydrolysis S_{ON} and S_{OP} begin to fall. S_S begins to rise, as heterotrophs are no longer utilizing it to grow, but does not increase substantially due to lack of hydrolysis from X_S. Nitrate levels rise as heterotrophs die, because the heterotrophic population had been using it as an N-source since S_{NH} dropped below 1mg/L. Autotrophic bacterial levels rise as they begin responding to the increased S_{NO3} levels.

On longer time scales, it is observed that the heterotrophs do not rebound:

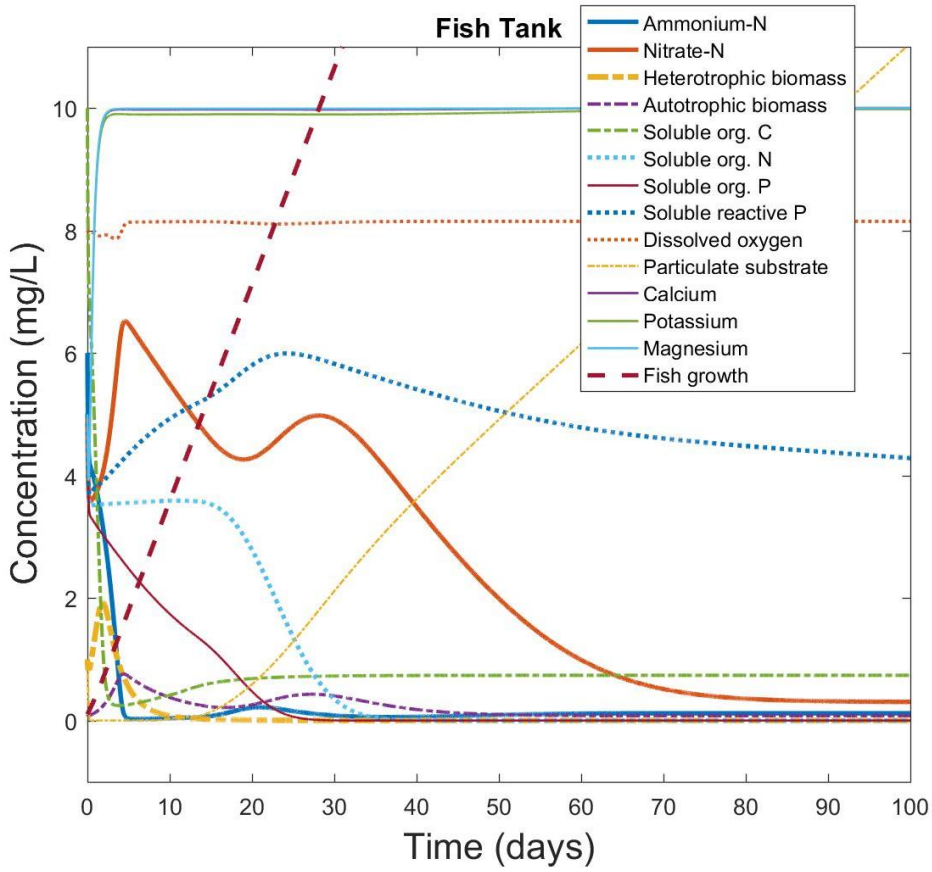

Figure 15. Fish tank output, 100 days

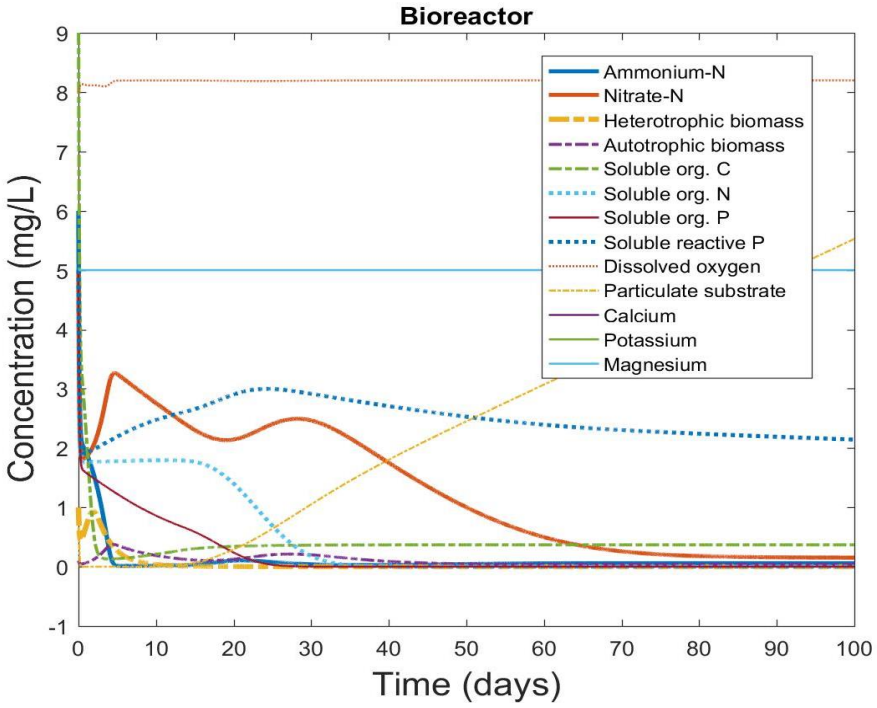

Figure 16. Fish tank output, 100 days

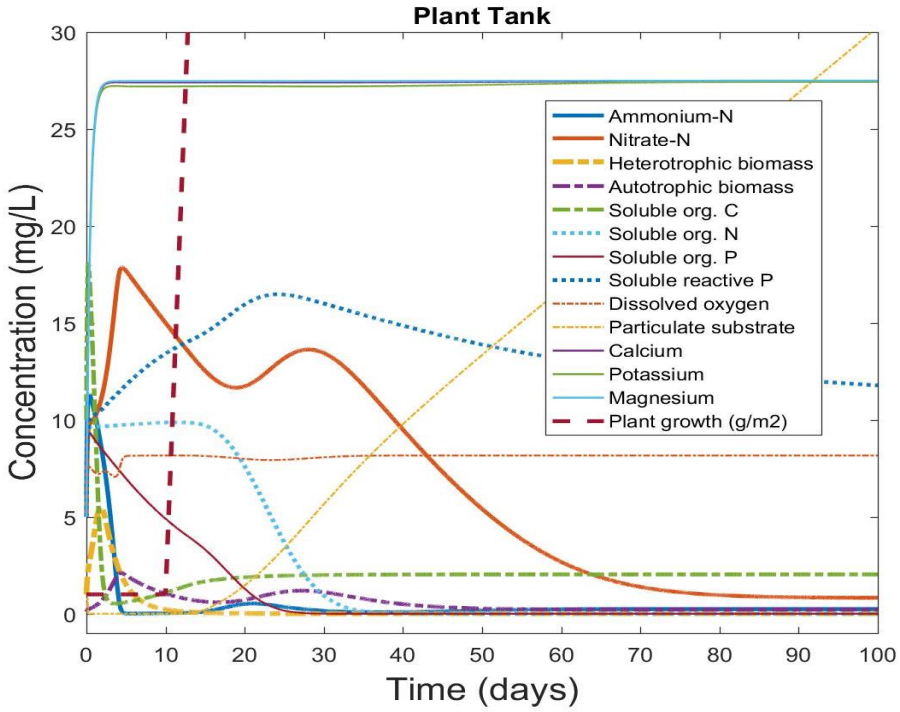

Figure 17. Plant tank output, 100 days

It is clear from the long-term results that there is a kinetic imbalance between magnitudes of heterotrophic growth and decay, with decay likely being too high for the bacteria to accumulate properly. Once levels of bacteria are low enough, hydrolysis ceases and prevents them from rebounding since S_S is also fairly low compared to the governing heterotrophic half-saturation coefficient ($K_{S_S,het} = 5\ mg/L$). As previously mentioned, the parameters used in this trial run should be carefully derived for an aquaponics system to avoid imbalances.

Table 9. Average concentrations of state variables, day 25

Variable	*Average value in all three tanks, day 25*	*Units*
X_{BH}	0.0047	mg/L
X_{BA}	0.3804	mg/L
S_S	1.0124	mg/L
X_S	1.7716	mg/L
S_{ON}	1.9114	mg/L
S_{NH}	0.1949	mg/L
S_{NO3}	6.7977	mg/L
S_{OP}	0.087	mg/L
S_{SRP}	8.483	mg/L
S_{O2}	8.0809	mg/L
$PlantDensity$	101	g/m^2
C_{fish}	68	mg/L
S_K	14.03	mg/L
S_{Ca}	14.13	mg/L
S_{Mg}	14.15	mg/L
X_I	108.08	mg/L

A majority of the ammonium and particulate substrates are removed by bacteria. The low levels of soluble organic carbon also indicate their consumption by heterotrophs is balanced with addition via dead bacteria and fish feces. S_{SRP} builds up to reasonably high

levels, which is beneficial for the plants. Calcium, magnesium, and potassium were all introduced at high levels in the initial conditions, resulting in their high concentrations at day 25; their uptake by plants is rapid (small half-saturation coefficient values), but plants do not require large amounts of them. The only major waste product from the system itself, besides the unmarketable portions of the fish and plant harvests, is the inert sludge. In this model, the sludge was not removed from the system, so it accumulates to unreasonably high concentrations, though in a real system with a solids removal mechanism this would not occur. According to the model, there should be 108.08 mg/L of inert material in the system water by the end of the 25-day trial. An error in the model code caused the rate of addition of inert mass to increase exponentially, so this term has instead been calculated separately according to the expected mass balance.

Plant growth appears rapid according to the results; this is likely due to the alteration of parameters to optimize for idealized growth and to exaggerate system kinetics. The excess of Ca, Mg, K, dissolved oxygen, and SRP all provide well for the plants' optimal growth.

It is clear from the resulting output graphs that the system will be N-limited over time (note the downward nitrate curve and very low ammonium levels). This is likely avoidable by balancing or dynamically changing the fish-to-plant ratio as needed. Hence, this limitation may not pose a significant problem once the model parameters are known more specifically for aquaponics systems.

5.3 Food harvest and nutrient transformation efficiency results

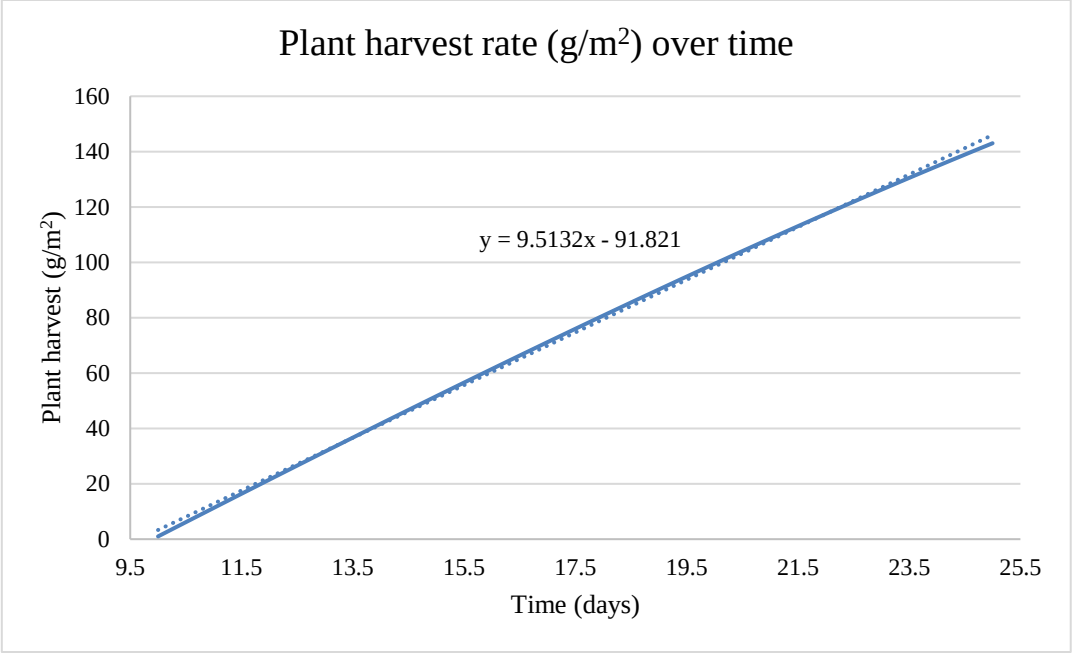

Figure 18. Plant harvest rate

Integrating a linear approximation of the plant harvest curve beginning on day 10, it was calculated that 164.8 kg (or 1119.9 g/m^2) (fresh weight) of lettuce was harvested over the course of 15 days.

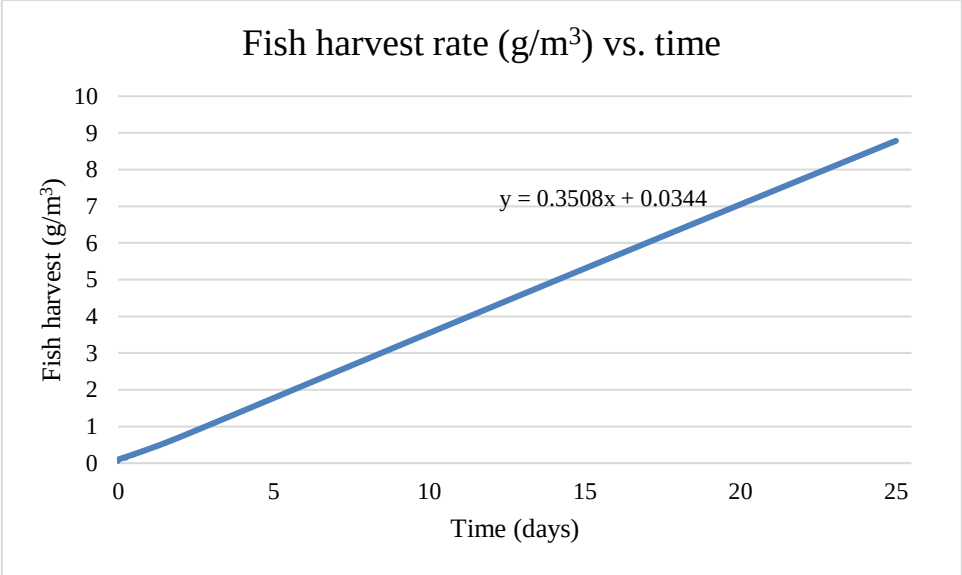

Figure 19. Fish harvest rate

By integrating the resulting fish growth curve, it was similarly calculated that 1.802 kg (or 110.485 g/m^3) (fresh weight) of tilapia was harvested over the course of 25 days.

5.4 Water balance results

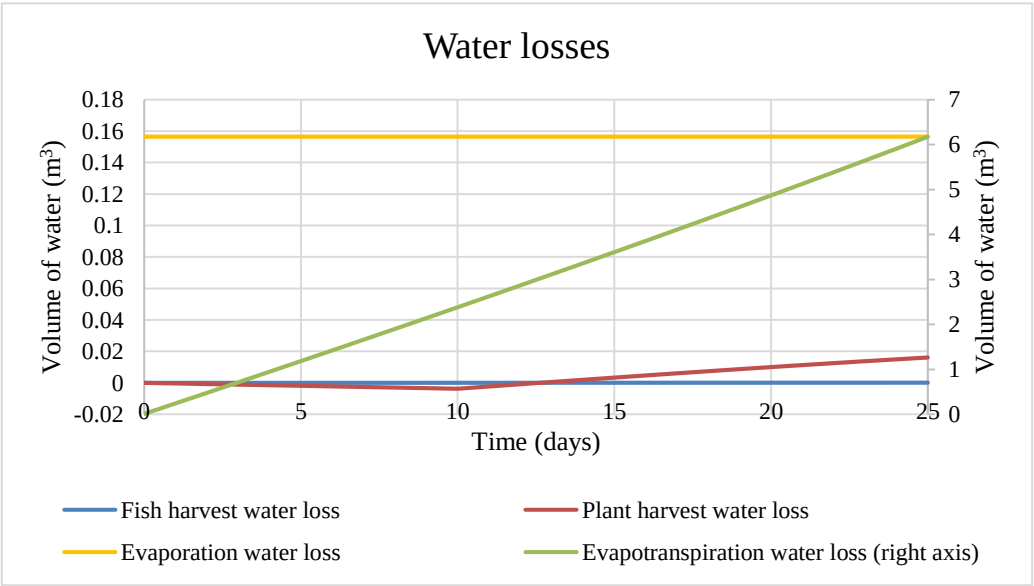

Figure 20. Water losses over time

The water balance results indicate that the most significant process contributing to water demand is evapotranspiration (see Figure 20). Over the course of the 25-day run, 75.1m^3 of total water was lost through evapotranspiration, equating to more than the entire system water volume of 69.32m^3. This is about 4.3 times higher than what is observed in aquaponic literature, which shows that about 1% of system water must be replaced each day[62]. This could indicate an error in the code, but may also be representative of the fact that the Penman-Monteith equation is applicable to soil-based crops rather than hydroponic ones: the soil itself contributes to the evaporative flux, whereas in this model the water surface is covered by Styrofoam rafts. Moving forward, new estimates of evapotranspiration may need to be derived specifically for these soilless raft systems.

Water losses due to removal of harvested fish and plants show expected trends, with plant water losses being negative before the plants are sown on day 10. Evaporation rate was estimated as a constant according to historical pan evaporation measurements for Atlanta.

5.4.1 Greenhouse air flow results

Total airflow Q_A depends on Q_{evap} and Q_{ET}. Due to the abnormally large levels of evapotranspiration and potentially an error in the code, the required airflow rate to keep greenhouse humidity levels constant was unrealistically high (about 10^9 m^3/day). This value assumes an outdoor temperature of 19°C and an indoor temperature of 24°C, with 70% humidity both inside and outside the greenhouse. This issue will be addressed in later iterations of the model code.

CHAPTER 6. DISCUSSION AND NEXT STEPS

The model is responsive in the expected ways to changes in the initial conditions or kinetic parameters. Extremely high plant densities, for example, deplete nitrate and soluble reactive phosphorus much more quickly than extremely low ones, and when any given nutrient becomes limiting, growth rate of the organism in question drops accordingly. The fish, plants, heterotrophs, and autotrophs are each differentially sensitive to changes in the nutrient composition of the food or water, respectively, as predicted by the model.

6.1 Food harvest productivity

The harvest rates for plants was low when compared to similar literature studies. An aquaponics system in Atlanta was capable of producing 134.2 g/(m^2-day) of leafy greens, equating to 3.35 kg/m^2 over 25 days or about 3 times the productivity shown here[6]. Initial density in this trial of the model was comparatively low, which may contribute to the discrepancy. Fish harvest rates, on the other hand, were higher than literature stocking density to harvest ratios suggest. By literature calculations, the fish harvest over 25 days was expected to have been only 368.9g, which is roughly 1/5 of the yield observed in this study[63]. The high fish harvest rate in this study is likely due to the omission of several other inhibitory kinetic terms within the extended Monod rate equation. Importantly, high stocking densities lower the growth rate of each fish, a term which was not included here and may have contributed to the high yield predictions.

6.2 Nutrient removal capacity

Understanding and optimizing the CEA's role as a water treatment technology could greatly enhance adoption of the technology. The mechanism of plant uptake and subsequent harvest removed 11% of the nitrogen, 19% of the phosphorus, 48% of the potassium, 22.6% of the calcium, and 26.7% of the magnesium that was added to the system via the fish food over the course of 25 days. This shows the significant potential these systems have for capturing and recycling nutrients. In later model iterations, standardized water quality parameters (e.g. carbonaceous BOD (CBOD), nitrogenous BOD (NBOD), total Kjeldahl nitrogen (TKN), total suspended and volatile solids (TSS and VSS), total dissolved solids (TDS), and others) will be used, which will allow direct comparisons with aquaculture effluent permits. This will facilitate direct scaling decisions for individual existing aquaculture systems or waste streams and aid in evaluating the CEA's water treatment capabilities.

6.3 Next steps and expansions

Due to the complexity of the system, some important aspects were not included in this iteration of the model, but their addition would increase the accuracy of predicted results. In future extensions, equations to model anaerobic processes such as denitrification and fermentation should be included. Micronutrients should also be considered. The next iteration of the model should separate fish and plants into categories based on size for better kinetic modeling purposes.

Several other aspects of fish and plant biology were not included due to their complexity, but should be included in future extensions. Separation of resource allocation to different phenotypic tissue types in fish and plants (i.e., stems, roots, fruits, and leaves

for plants, and muscle, bone, and skin in fish) would allow a more precise prediction of harvest capability based on crop and fish species. A leafy green such as spinach has a higher marketable percentage than a fruiting crop such as a tomato, for example. Similarly, 'luxury' nutrient uptake by plants when they have access to excess available nutrients was not included, but could play an important role in the overall nutrient removal capacity of the system.

6.3.1 Economic analysis and LCA

The economic analysis of a CEA should be among the next studies conducted using the results from this model. Adding a dynamic cost calculation could accurately predict energy and operations costs and harvest profits to determine the most economically-viable scale and operation scheme. This could be further expanded to include location-specific building and transport costs according to the local urban density and property prices.

Similarly, a life-cycle-assessment (LCA) of a CEA would also be more readily calculable using results from this model. This type of study would allow an analysis of differential benefits to locally-grown food vs. conventionally farmed food over its entire life cycle, and would be the determining factor in overall decision-making regarding the sustainability of a CEA in the long term. While the social and economic benefits of CEAs may be obvious, the emissions associated with the local energy mix used to build and power these systems combined with the relatively small scale of production may outweigh the environmental benefits.

6.3.2 Urban ecological analysis and scale-up

In the context of urban ecology, CEAs could act as hubs for nutrient removal from waste and wastewater and simultaneously as a center for food production. In conjunction with other models for the location and quality of urban nutrient streams and geographic information systems (GIS) analysis of available, unshaded land area, this model could be used to predict the ideal scale, location, and per-capita efficiency of waste nutrient recycling and food production for a given urban neighborhood. Work in this area is already underway at Georgia Tech[64].

With this information comes the power to make important decisions about where and how to implement this new technology. This can help researchers, city planners, businesses, and policymakers answer questions about the feasibility of relying on CEAs and waste-grown algae to treat wastewater and provide food for a given area's population. The model could be further leveraged to determine ideal locations for points of sale for the produce generated in the CEA to maximize productivity depending on population density, and to further minimize transportation distance between food growth and consumption. Existing potentially useful infrastructures such as municipal solid waste pickup service routes, sewage pipes and sumps, power plants, and food processing facilities can all be considered with regard to their relative importance in the siting and scaling of CEAs in future combined modeling efforts. For example, one large CEA, perfectly situated to make use of waste heat from a power plant and waste nutrients from a food processing facility, may be less productive in terms of food generated or distance to points-of-sale, but overall more energetically sustainable than several smaller CEAs each with less than ideal nutrient and power inputs.

6.4 Influence of the model in policy and food security

As an up and coming industry, many would-be aquaponics growers are intimidated by the complexity of combining two mature and complex industries: hydroponics and aquaculture. Initial investments can be high, with hypothesized yields and profit margins based on empirical fish:plant ratios. Errors in nutrient balances may harm fish or cause nutrient deficiency in plants, or vice versa. This precarious balance is not favored by farmers; especially for the small-scale grower, there is a perceived difficulty to entry into this highly efficient method of agriculture. With the help of this model and its future iterations, a tailored CEA system can be built to fit the client's physical space limitations, yields can be predicted to a higher degree of accuracy for both fish and plant crops, and exact nutritional and water needs can be calculated over time and for all types of climactic conditions throughout the year.

The CEA's ability to recycle and productively use existing nutrient, energy, and water flows based on existing urban infrastructure makes it a promising step forward in sustainable urban technology. Cities with significant local food production capacity will be more self-sufficient and resilient in the face of climate change. Making this technology more accessible through robust, mechanistic models will inspire confidence in this new type of agriculture, allowing the CEA to become an important part of the sustainable future.

APPENDIX A.

A.1 MATLAB model code

A.1.2 Main model code

```
function [FISHBIOPLANTS]=Fishbioplants9g(t,species)

%SYSTEM PARAMETERS
%-----Fish-----
      %Fish tank: environmental characteristics
A_fish = 13.378; %Top surface area of the fish tank (m^2)
d_fish = 1.2192; %depth of the fish tank, assumed constant (m)
V_fish = 16.31; %A_fish .* d_fish; %volume of fish tank, m^3
numfish = 50.*V_fish; %assuming 50 fish/m3, approximate ideal
stocking density
Tw = 27.284; %degrees C
K_I_Tw = 39.1876; %Haldane coefficient for water temp. dependence
k_La = 22; %ALTERED, LIT SAYS 2 oxygen mass transfer coefficient,
1/days
S_O2_sat = 8.2; %g/m3 saturation of oxygen at 24C in water
      %Fish: kinetic parameters
b_fish = 0.001497; %fish fasting catabolism rate (g^(1-0.81)/d)
K_F_C = 0.1;%0.22; %half sat. coefficient for food-C dependence
K_F_Ca = 0.005;%0.011; %half-sat. coeff. for food Ca dependence
K_F_K = 0.009493; %Half sat. coefficient for food-K dependence
K_F_Mg = 0.0003715; %half-sat. coeff. for food Mg dependence
K_F_N = 0.01;%0.03; %Half sat. coefficient for food-N dependence
K_F_P = 0.00233; %Half sat. coefficient for food-P dependence
k_fish = 0.02;%max substrate use rate = feed rate in this case.
K_SO2_fish = 0.898; %half sat. coefficient for DO dependence
K_Ss_fish = 30; %half sat. coefficient for Ss dependence
K_Tw = 18.9963; %half sat. coefficient for water temp. dependence
Y_fish = 0.6952; %g fish/g food
      %Fish tissue: mass characterization
i_N_fish = 0.04;%0.06;%0.0979; %fraction of N in fish tissue (g
N/g fish)
i_water_fish = 0.8; %fraction of water in harvested fish tissue
i_C_fish = 0.4; %fraction of C in fish tissue (
i_P_fish = 0.02; %
i_K_fish = 0.03; %
i_Ca_fish = 0.02; %
i_Mg_fish = 0.005;
      %Fish feed: mass characterization
F_C = 0.5; %fraction of C in fish food (g C/g food)
F_Ca = 0.051; %fraction of Ca in fish food (g Ca/g food)
F_K = 0.032; % fraction of K in fish food (g K/g food)
```

```
F_Mg = 0.003545; %fraction of Mg in fish food (g Mg/g food)
F_N = 0.2; %fraction of N in fish food (g N/g food)
F_P = 0.022; %fraction of P in fish food (g P/g food)
% F_inert = 0.458455; %(1-sum of all other fractions)
      %Fish feces: mass characterization
i_Ca_feces = 0.0699; %Moccia et. al
i_deg_feces = 0.8; %fraction of degradable material in feces
i_K_feces = 0.001; %Moccia et. al
i_Mg_feces = 0.0053; %Moccia et. al
i_N_feces = 0.0295; %(or: 0.46 .* F_N) %N content in feces
i_P_feces = 0.022; %
i_C_feces = 0.4;

%----------Bacteria----------
      %Bacteria: Kinetic parameters
b_A = 0.15; %autotrophic decay rate (1/day)
f_d = 1;%0.8; %fraction of degradable material in biomass
(approximation from Dr. P's notes)
k_A = 4.166; %autotrophic max. substrate use coefficient (g NH
used/g X_BA formed-day)
K_NH_A = 1; %autotrophic 1/2 sat. coefficient for NH dependence
(g NH/m^3)as NH4+
K_SO2_A = 0.5; %auto. 1/2 sat. coefficient for oxygen dependence
(g SO2/m^3)
K_SRP_A = 0.01; %auto. 1/2 sat. coefficient for SRP dependence (g
SRP/m^3)
u_A = 1; %autotrophic max. specific growth rate (1/day)
Y_A = 0.24; %autotrophic yield (g autotrophic biomass formed/g NH
used)
b_H =  0.62; %heterotrophic decay rate (1/day)
k_het = 2.766; %het max. substrate use coefficient (g S_S used/g
X_BH formed-day)
K_NH_het = 0.05; %het 1/2 sat. coefficient for NH dependence (g
NH/m^3)as NH4+
K_SO2_het = 0.2; %het1/2 sat. coefficient for oxygen dependence
(g DO/m^3)
K_SRP_het = 0.01; %het 1/2 sat. coefficient for SRP dependence (g
SRP/m^3)
K_SS_het = 5;%20; %het 1/2 sat. coefficient for COD substrate
dependence (g COD_s/m^3)
u_H = 2.13; %heterotrophic max. specific growth rate (1/day)
Y_H = 0.77; %heterotrophic yield (g X_BH formed/g S_S used)
      %Hydrolysis and transformation kinetic parameters
K_H_SO2 = 0.2; %1/2 sat. coefficient for oxygen dep in hydrolysis
of X_S (g SO2/m^3)
K_H_Xs = 0.001;%0.1;%0.0102; %1/2 sat. coefficient for hydrolysis
of particulate substrate
k_H_Xs = 5;%3;%1.14; %maximum specific hydrolysis rate for
particulate COD (g X_S/g X_BH -day)
k_a = 0.08; %.
%ammonification rate (1/d)
k_m = 0.22;%. mineralization rate (1/d)
```

```
      %Bacteria: Mass characterization
i_N_bio = 0.117; %fraction of N in bacterial biomass (g N/g
biomass)
i_P_bio = 0.052; %fraction of P in bacterial biomass (g P/g
biomass)
i_C_bio = 0.53; %
i_C_Xs = (i_C_bio + i_C_feces)./2;
i_P_Xs = (i_P_bio + i_P_feces)./2;
i_N_Xs = (i_N_bio + i_N_feces)./2;
%i_inert_Xs = (1-f_d) + 0.1914;
      %Bioreactor physical characteristics:
A_bioreactor = 16.31; %Top surface area of bioreactors (m^2)
d_bioreactor = 0.5; %depth of bioreactor (m)
V_bioreactor = A_bioreactor.*d_bioreactor; %bioreactor volume
(sum of all 6, no media displacement assumed) (m^3)
S_O2_sat = 8.2; %g/m3 saturation of oxygen at 24C in water

%----------Plants------------
      %Plants: environmental characteristics
A_plant = 147.1584; %top surface area of the plant tank (m^2)
d_plant = 0.3048; %depth of the plant tank (m)
V_plant = 44.8538; %A_plant .* d_plant; %volume of the water in
the plant tank (m^3)
numplant = 25.*A_plant; %assume 8 inches or 0.2m apart
S_O2_sat = 8.2; %g/m3 saturation of oxygen at 24C in water
P_CO2 = 1.*10^-3.42; %atm (double check units here)
%T_GH = 24; %
      %Plants: Kinetic parameters
b_plant = 0.0265; %plant respiration rate (1/day)
K_Ca_plant =1; %plant 1/2 sat. coefficient for calcium dependence
K_I_PPFD = 1600; %Haldane coefficient for light intensity
dependence
K_I_TGH_plant = 31; %Haldane coeff for air temperature dependence
K_I_Tw = 39.1876; %Haldane coefficient for water temp. dependence
K_K_plant = 1;%7.22; %plant 1/2 sat. coeff for potassium
K_Mg_plant = 0.243; %plant 1/2 sat. coeff for Magnesium
K_NH_plant = 10; %plant 1/2 sat. coefficient for ammonia
K_NO3_plant = 4;%10; %plant 1/2 sat. coefficient for nitrate
K_PCO2_plant = 3e-4;%2e-4; %plant 1/2 sat. coeff for atm CO2
dependence
k_plant = 6.526;%maximum subst use plants
K_PPFD = 200; %plant 1/2 sat. coefficient for light intensity
dependence(umol/m^2-s)
K_SO2_plant = 5; %plant 1/2 sat. coefficient for DO dependence
(mg/L)
K_SRP_plant = 3; plant 1/2 sat. coefficient for P dep. (g/m^3)
K_TGH_plant = 17; %plant 1/2 sat. coefficient for atmospheric
temperature dependence
u_plant = 1.3; %g/m2-day %max specific plant growth rate (1/day)
Y_plant = 0.1992; % (g dry plant added/g bioavailable nutrients
taken up)
PPFD = 1500;
```

```
      %Plant tissue: mass characterization
i_C_plant = 0.337; % fraction of C in plant mass (g C/g dry
plant)
i_Ca_plant = 0.0122; %fraction of Ca in plant mass (g Ca/g dry
plant)
i_K_plant = 0.0390; % fraction of K in plant mass (g K/g dry
plant)
i_Mg_plant = 0.0036; %fraction of Mg in plant mass (g Mg/g dry
plant)
i_N_plant = 0.0584; %fraction of N in plant mass (g N/g dry
plant)
i_P_plant = 0.0104; % fraction of P in plant mass (g P/g dry
plant)
i_water_plant = 0.95; %fraction of water in fresh plant mass

%-----------------------------------
%-----------------------------------
      %Evapotranspiration
T_GH = 24; %degrees C
Patm = 101.325; %
p_GH = (Patm.*1000)./(287.058.*(T_GH+283.15)); %density of air in
greenhouse (g/m3)
e_s_GH = 0.6108.*exp((17.27.*27)./(27+237.3)); %saturation vapor
pressure at T_GH (27C).
u_z = 1.5; %wind speed, ESTIMATE (m/s)
T_out = 19; %Temperature of the air outside the greenhouse (C)
omega_ext = 10./1000; %(g H2O/g air)Humidity ratio of air outside
the greenhouse (g H20/g dry air) at T_out
omega_GH = 14./1000; %(g H2O/g air) humidity ratio of air inside
greenhouse
p_ext = (Patm.*1000)./(287.058.*(T_out+283.15)); %density of air
outside greenhouse (g/m3)
DEG = 33;
Tw = 24;
T_max = 30;
T_min = 20;
T_maxK4 =41.41;
T_minK4 =36.21;
P_atm = 101.325; %air pressure
S_O2_sat = 8.2; %g/m3 saturation of oxygen at 24C in water
alpha = 0.2;
delta =
(4096.*(0.6108.*exp((17.27.*T_GH)./(T_GH+237.3)))./((T_GH+237.3)
^2);%
phi = (pi./180).*DEG; %latitude in radians
p_a = P_atm./(287.058.*(T_GH+283.15)); %mean air density
c_p = 0.001013; %specific heat of air
lambda = 2.45; %latent heat of vaporization of water
e = 0.622; %ratio of MW of water to dry air
gamma = (c_p.*P_atm)./(e.*lambda); %psychrometric constant, kPa/C
h = 2;
z_m = 2;
```

```
k = 0.41; %Von Karman's constant
z = h; %height of ET measurement
d0 = (2./3).*h; % zero plane displacement height
z_h = z_m; %height of humidity measurements
z_om = 0.123.*h; %roughness length for momentum transfer
z_oh = 0.1.*z_om; %roughness length for transfer of heat/vapor
u_z = 1.5; %m/s (avg wind speed, chosen arbitrarily)
r_a = (log((z_m-d0)./z_om).*log((z_h-d0)./z_oh))./(k^2.*u_z);
%aerodynamic resistance
LAI = 1; %assuming 100% transpiration no evaporation from surface
LAI_active = 1; %
r_i = 70; % estimated bulk stomatal resistance, well-lit leaf
r_s = r_i./LAI_active; %bulk surface resistance
e_s = 0.6108.*exp((17.27.*T_GH)./(T_GH+237.3));%
e_a =0.7.*e_s;
sigma = (4.903.*10^-9); %stefan-boltzmann constant
G_sc = 0.082; %solar constant
a_s = 0.25;
b_s = 0.5;
G = 0; %soil heat flux, small compared to R_n
J = ceil(t);
d = 0.409.*sin(((2.*pi.*J)/365)-1.39); %lowercase delta
d_r = 1+0.033.*cos((2.*pi.*J)./365);
omega_s = acos(-tan(phi).*tan(d)); %sunset hour angle
N = (1./pi).*omega_s; %total timespan of sunlight on day number J
(in days)
R_a
=458.3662.*G_sc.*d_r.*(omega_s.*sin(phi).*sin(d)+cos(phi).*cos(d)
.*sin(omega_s));%
R_s = (a_s + b_s).*R_a;
R_nl = sigma.*((T_maxK4 + T_minK4)./2).*(0.34-
0.14.*sqrt(e_a)).*(1.35-0.35);
R_ns = (1-alpha).*R_s;
R_n =R_ns - R_nl; %net solar radiation contributing to final ET
equation
R_nmax = 10;
%-------------------------------------------------------------------
                  %FLOW STATE VARIABLES
                  Q_P = species(60);
                  Q_H = species(61);
                  Q_A = species(62);
                  Water = species(63);
                  Q_ET = species(64);
%-----------------------------------------------------------
            %Flow characterization

                Q0 = 100;   %m3/day, flow in between all tanks
                Q_evap = 0.1565;%<--m/d.
                 %evapotranspiration in m3/day
Q_ET =((delta.*R_n-G +p_a.*c_p.*((e_s-e_a)./r_a))./(delta +
lambda.*gamma.*(1+(r_s./r_a)))).*(A_plant./1000);
              %ET ESTIMATE: 3mm/day or 0.44145 m3/d
```

```
                Q_in_tapf = Q_H + Q_evap;
                Q_in_tapp = Q_P + Q_ET;
                Q_f = Q0 - Q_evap - Q_H + Q_in_tapf;
                Q_p = Q0 - Q_ET - Q_P + Q_in_tapp;
                Q_b = Q0;
                Q_in_tap = Q_in_tapf + Q_in_tapp;
                Q_A = ((-Q_ET-Q_evap)./(p_ext.*omega_ext -
p_GH.*omega_GH))./(10^-6);%

%-----------------------------------------------------------------
%FISH TANK STATE VARIABLES
    S_Sf = species(1); %readily biodegradable substrate
    X_Sf = species(2); %slowly biodegradable (particulate)
substrate
    S_If = species(3); %soluble inert material
    X_BHf = species(4); % active heterotropic biomass
    X_BAf = species(5); % active autotrophic biomass
    S_NHf = species(6) ; %ammonia nitrogen
    S_NO3f = species(7); % nitrate nitrogen
    S_ONf = species(8); % soluble organic nitrogen
    S_O2f = species(9); % oxygen concentration
    C_fishf = species(10); %fish
    S_OPf = species(11); %soluble organic phosphorus
    S_SRPf = species(12); %soluble reactive phosphorus
(orthophosphates)
    S_Kf = species(13); %free potassium ion
    S_Mgf = species(14); %free magnesium ion
    S_Caf = species(15); %free calcium ion
    Hf = species(16); %Fish harvest rate (g/day)
    Rf = species(17); %Fish feeding rate (g/day)
    Xi_f = species(18);
    Cf = species(19);
    Nf = species(20);
    Pf = species(21);
            Mfish = C_fishf.*V_fish;
%BIOREACTOR STATE VARIABLES
    S_Sb = species(22); %readily biodegradable substrate
    X_Sb = species(23); %slowly biodegradable (particulate)
substrate
    S_Ib = species(24); %soluble inert material
    X_BHb = species(25); % active heterotropic biomass
    X_BAb = species(26); % active autotrophic biomass
    S_NHb = species(27) ; %ammonia nitrogen
    S_NO3b = species(28); % nitrate nitrogen
    S_ONb = species(29); % soluble organic nitrogen
    S_O2b = species(30); % oxygen concentration
    S_OPb = species(31); %soluble organic phosphorus
    S_SRPb = species(32); %soluble reactive phosphorus
    S_Kb = species(33); %free potassium ion
    S_Mgb = species(34); %free magnesium ion
    S_Cab = species(35); %free calcium ion
    Xi_b = species(36);%inert material in the bioreactor
```

```
    Cb = species(37);
    Nb = species(38);
    Pb = species(39);
%PLANT TANK STATE VARIABLES
    S_Sp = species(40); %readily biodegradable substrate
    X_Sp = species(41); %slowly biodegradable (particulate)
substrate
    S_Ip = species(42); %soluble inert material
    X_BHp = species(43); % active heterotropic biomass
    X_BAp = species(44); % active autotrophic biomass
    S_NHp = species(45) ; %ammonia nitrogen
    S_NO3p = species(46); % nitrate nitrogen
    S_ONp = species(47); % soluble organic nitrogen
    S_O2p = species(48); % oxygen concentration
    S_OPp = species(49); %soluble organic phosphorus
    S_SRPp = species(50); %soluble reactive phosphorus
    S_Kp = species(51); %free potassium ion
    S_Mgp = species(52); %free magnesium ion
    S_Cap = species(53); %free calcium ion
    PlantDensityp = species(54); %plant mass per area (g/m2)
    Pp = species(55); %plant harvest rate (g/day)
    Xi_p = species(56); %inert material in the plant tank
    Cp = species(57);
    Np = species(58);
    Pplant = species(59);

%--------------------------------------------------------------------
%--------------------------------------------------------------------

%FISH TANK RATE EQUATIONS
            %Fish food input rate
    r_food = 0.02.*Mfish; %(g food/d)
            %Fish growth rate (1/d)
    r_fishf = Y_fish.*0.02.*(F_C ./ (K_F_C + F_C)) .* (F_P ./
(K_F_P + F_P)) .* (F_N ./ (K_F_N + F_N)) .* (F_K ./ (K_F_K +
F_K)) .* (F_Ca ./ (K_F_Ca + F_Ca)) .* (F_Mg ./ (K_F_Mg + F_Mg))
.* (S_O2f ./ (K_SO2_fish + S_O2f)) .* (K_Ss_fish ./ (K_Ss_fish
+S_Sf));% .*(Tw ./ (K_Tw + Tw + (Tw^2 ./ K_I_Tw) ));
            %Fish carbon exhalation rate
    r_fishresp = 0.02.*0.3.*Mfish.*F_C; %(gC/d)

    if S_NHf >= 1
            %Heterotrophic growth rate (1/d)
    r_Hf = k_het.*Y_H.*(S_O2f ./ (K_SO2_het +S_O2f)).* (S_Sf ./
(K_SS_het + S_Sf) ) .* (S_NHf ./ (K_NH_het + S_NHf)) .* (S_SRPf
./ (K_SRP_het + S_SRPf));
    else
            %Heterotrophic growth rate (1/d)
    r_Hf = k_het.*Y_H.*(S_O2f ./ (K_SO2_het +S_O2f)).* (S_Sf ./
(K_SS_het + S_Sf) ) .* (S_NO3f ./ (K_NH_het + S_NO3f)) .* (S_SRPf
./ (K_SRP_het + S_SRPf));
    end
```

```
            %Autotrophic growth rate (1/d)
    r_Af = k_A.*Y_A.*(S_O2f ./ (K_SO2_A + S_O2f)) .* (S_NHf ./
(K_NH_A + S_NHf)) .* (S_SRPf ./ (K_SRP_A + S_SRPf));
            %Heterotrophic carbon exhalation rate (gC/d)
    r_Hrespf =r_Hf.*X_BHf.*1.947.*(12./44).*V_fish;
            %Heterotrophic decay rate (1/d)
    %r_hetdecay = b_H;
            %Autotrophic decay rate (1/d)
    %r_Adecay = b_A;
            %Fish mass loss rate due to basline metabolic
catabolism (g fish/d)
    r_catabolism = b_fish.*(Mfish)^0.81;
            %Fish gill nitrogen excretion rate (gN/d)
    r_gillN = 0.02.*Mfish.*F_N.*0.8; %changed from 39% of food-N
            %Fish feces CARBON excretion rate (gC/d)
    r_fecesC = r_food.*F_C - r_fishf.*i_C_fish.*Mfish -
r_fishresp - r_catabolism.*i_C_fish;
            %Fish feces NITROGEN excretion rate (gN/d)
    r_fecesN = r_food.*F_N - r_fishf.*i_N_fish.*Mfish - r_gillN -
r_catabolism.*i_N_fish;
            %Fish feces PHOSPHORUS excretion rate
    r_fecesP = r_food.*F_P - r_fishf.*i_P_fish.*Mfish;
            %Fish feces K excretion rate
    r_fecesK = r_food.*F_K - r_fishf.*i_K_fish.*Mfish;
            %Fish feces Ca excretion rate
    r_fecesCa = r_food.*F_Ca - r_fishf.*i_Ca_fish.*Mfish;
            %Fish feces Mg excretion rate
    r_fecesMg = r_food.*F_Mg - r_fishf.*i_Mg_fish.*Mfish;
            %Fish feces INERT mass excretion rate
    r_fecesinert =
0.111.*(r_fecesC+r_fecesN+r_fecesP+r_fecesK+r_fecesCa+r_fecesMg);

            %Hydrolysis rate (gXs/d)
    r_hydrolysisf = k_H_Xs .* (S_O2f ./ (K_H_SO2 + S_O2f)).*
(((X_Sf ./ X_BHf)./(K_H_Xs + (X_Sf ./ X_BHf))).*X_BHf).*V_fish;
            %Aeration rate (g O2/d)
    r_aerationf = k_La .* (S_O2_sat - S_O2f).*V_fish;
            %Fish harvest rate (g fish/d)
    r_fish_harvest = r_fishf.*Mfish - r_catabolism;

%BIOREACTOR RATE EQUATIONS (any unique ones not present above)
    if S_NHb >=1
            %Heterotrophic growth rate (1/d)
    r_Hb = k_het.*Y_H.*(S_O2b ./ (K_SO2_het +S_O2b)).* (S_Sb ./
(K_SS_het + S_Sb) ) .* (S_NHb ./ (K_NH_het + S_NHb)) .* (S_SRPb
./ (K_SRP_het + S_SRPb));
    else
       %Heterotrophic growth rate (1/d)
    r_Hb = k_het.*Y_H.*(S_O2b ./ (K_SO2_het +S_O2b)).* (S_Sb ./
(K_SS_het + S_Sb) ) .* (S_NO3b ./ (K_NH_het + S_NO3b)) .* (S_SRPb
./ (K_SRP_het + S_SRPb));
    end
```

```
        %Autotrophic growth rate (1/d)
    r_Ab = k_A.*Y_A.*(S_O2b ./ (K_SO2_A + S_O2b)) .* (S_NHb ./
(K_NH_A + S_NHb)) .* (S_SRPb ./ (K_SRP_A + S_SRPb));
            %Heterotrophic carbon exhalation rate (gC/d)
    r_Hrespb =r_Hb.*X_BHb.*1.947.*(12./44).*V_bioreactor;

            %Hydrolysis rate (gXs/d)
    r_hydrolysisb = k_H_Xs .* (S_O2b ./ (K_H_SO2 + S_O2b)).*
(((X_Sb ./ X_BHb)./(K_H_Xs + (X_Sb ./
X_BHb))).*X_BHb).*V_bioreactor;

             %Aeration rate (g O2/d)
    r_aerationb = k_La .* (S_O2_sat - S_O2b).*V_bioreactor;

%PLANT TANK RATE EQUATIONS (any unique ones not present above)
    if S_NHp >= 1
            %Heterotrophic growth rate (1/d)
    r_Hp = k_het.*Y_H.*(S_O2p ./ (K_SO2_het +S_O2p)).* (S_Sp ./
(K_SS_het + S_Sp) ) .* (S_NHp./ (K_NH_het + S_NHp)) .* (S_SRPp ./
(K_SRP_het + S_SRPp));
    else
       %Heterotrophic growth rate (1/d)
    r_Hp = k_het.*Y_H.*(S_O2p ./ (K_SO2_het +S_O2p)).* (S_Sp ./
(K_SS_het + S_Sp) ) .* (S_NO3p./ (K_NH_het + S_NO3p)) .* (S_SRPp
./ (K_SRP_het + S_SRPp));
     end
        %Autotrophic growth rate (1/d)
    r_Ap = k_A.*Y_A.*(S_O2p ./ (K_SO2_A + S_O2p)) .* (S_NHp ./
(K_NH_A + S_NHp)) .* (S_SRPp ./ (K_SRP_A + S_SRPp));
            %Heterotrophic carbon exhalation rate (gC/d)
    r_Hrespp =r_Hp.*X_BHp.*1.947.*(12./44).*V_plant;

            %Hydrolysis rate (gXs/d)
    r_hydrolysisp = k_H_Xs .* ((S_O2p ./ (K_H_SO2 + S_O2p)).*
((X_Sp ./ X_BHp) ./ (K_H_Xs+(X_Sp./X_BHp)) ).*X_BHp).*V_plant;

            %Plant growth rate (1/d)
    r_plant = k_plant .* Y_plant .* (PPFD ./ (K_PPFD + PPFD
+(PPFD^2 ./ K_I_PPFD))) .* (S_NO3p ./ (K_NO3_plant +S_NO3p))
.*(P_CO2 ./ (K_PCO2_plant + P_CO2)).* (S_O2p ./(K_SO2_plant +
S_O2p))  .* (S_Kp./ (K_K_plant +S_Kp)) .* (S_Cap ./ (K_Ca_plant +
S_Cap)) .* (S_Mgp ./ (K_Mg_plant + S_Mgp)) .* (S_SRPp ./
(K_SRP_plant + S_SRPp)).* (T_GH ./ (K_TGH_plant + T_GH + (T_GH^2
./ K_I_TGH_plant)));
                    %---removed from plants: (S_NHp ./
(K_NH_plant + S_NHp))
            %plant respiration (1/d)
    %r_plantresp = 0.0265; %

            %plant harvest (1/d)
    r_plant_harvest = r_plant - b_plant; %
```

```
            %Aeration rate (g O2/d)
    r_aerationp = k_La .* (S_O2_sat - S_O2p).*V_plant;
%-----------------------------------------------------------------
%-----------------------------------------------------------------

%FISH TANK ODEs (g/m3-time)
    dS_Sf = (Q_p.*S_Sp)./V_plant  - (Q_f.*S_Sf)./V_fish + (r_Hf.*
(-1 ./ Y_H).*X_BHf.*V_fish + r_hydrolysisf.*i_C_Xs-
r_Hf.*1.184.*X_BHf.*V_fish)./V_fish;
    dX_Sf =(Q_p.*X_Sp)./V_plant - (Q_f.*X_Sf)./V_fish+
(b_H.*(f_d).*X_BHf.*V_fish + b_A.*(f_d).*X_BAf.*V_fish +
r_fecesC+r_fecesN+r_fecesP+r_fecesK+r_fecesCa+r_fecesMg -
r_hydrolysisf)./V_fish;
    dS_If = 0;
    dX_BHf = (Q_p.*X_BHp)./V_plant - (Q_f.*X_BHf)./V_fish +
(r_Hf.*X_BHf.*V_fish - b_H.*X_BHf.*V_fish)./V_fish;
    dX_BAf = (Q_p.*X_BAp)./V_plant - (Q_f.*X_BAf)./V_fish +
(r_Af.*X_BAf.*V_fish - b_A.*X_BAf.*V_fish)./V_fish;
    dS_NHf = (Q_p.*S_NHp)./V_plant - (Q_f.*S_NHf)./V_fish +
(r_Hf.*(-i_N_bio).*X_BHf.*V_fish)./V_fish + (r_Af.*(-
i_N_bio.*X_BAf.*V_fish) - r_Af.*(1./Y_A).*X_BAf.*V_fish + r_gillN
+ k_a.*S_ONf.*X_Sf.*V_fish)./V_fish;
    dS_NO3f = (Q_p.*S_NO3p)./V_plant- (Q_f.*S_NO3f)./V_fish +
(r_Af.*(1./Y_A).*X_BAf.*V_fish)./V_fish;
    dS_ONf = (Q_p.*S_ONp)./V_plant- (Q_f.*S_ONf)./V_fish +
(r_hydrolysisf.*i_N_Xs - k_a.*S_ONf.*X_Sf.*V_fish)./V_fish;
    dS_O2f = (r_Hf.*((-1-Y_H)./Y_H).*X_BHf.*V_fish + r_Af.*((-
4.57-Y_A)./Y_A).*X_BAf.*V_fish + r_fishf.*((-1-
Y_fish)./Y_fish).*C_fishf.*V_fish + r_aerationf)./V_fish;
    %(Q_p.*S_O2p)./V_plant- (Q_f.*S_O2f)./V_fish +
    dC_fishf =0;%(C_fishf.*V_fish.*(r_fishf - r_catabolism -
r_fish_harvest))./V_fish;
    dS_OPf =(Q_p.*S_OPp)./V_plant- (Q_f.*S_OPf)./V_fish +
(r_hydrolysisf.*i_P_Xs - k_m.*S_OPf.*V_fish)./V_fish;
    dS_SRPf =(Q_p.*S_SRPp)./V_plant- (Q_f.*S_SRPf)./V_fish
+(r_Hf.*(-i_P_bio).*X_BHf.*V_fish + r_Af.*(-
i_P_bio).*X_BAf.*V_fish + k_m.*S_OPf.*V_fish)./V_fish;
    dS_Kf = (Q_p.*S_Kp)./V_plant- (Q_f.*S_Kf)./V_fish+
(r_fecesK)./V_fish;
    dS_Mgf =(Q_p.*S_Mgp)./V_plant- (Q_f.*S_Mgf)./V_fish +
(r_fecesMg)./V_fish;
    dS_Caf =(Q_p.*S_Cap)./V_plant- (Q_f.*S_Caf)./V_fish +
(r_fecesCa)./V_fish;
    dCf = r_food.*F_C - r_fishf.*C_fishf.*V_fish.*i_C_fish -
r_fishresp - r_Hf.*X_BHf.*V_fish.*i_C_bio - r_Hrespf +
b_H.*X_BHf.*V_fish.*i_C_bio + b_A.*X_BAf.*V_fish.*i_C_bio
+r_catabolism.*i_C_fish;
    dNf = r_food.*F_N - r_fishf.*C_fishf.*V_fish.*i_N_fish +
r_gillN - r_Hf.*X_BHf.*V_fish.*i_N_bio -
r_Af.*X_BAf.*V_fish.*i_N_bio +r_fecesN +
b_H.*X_BHf.*V_fish.*i_N_bio + b_A.*X_BAf.*V_fish.*i_N_bio +
r_catabolism.*i_N_fish;
```

```
    dPf = r_food.*F_P - r_fishf.*C_fishf.*V_fish.*i_P_fish -
r_Hf.*X_BHf.*V_fish.*i_P_bio - r_Af.*X_BAf.*V_fish.*i_P_bio;
    dRf = (0.02.*Mfish)./V_fish;
    dHf = r_fish_harvest./V_fish;
    dXi_f = (Q_p.*Xi_p)./V_fish- (Q_f.*Xi_f)./V_fish+
0.1914.*r_food+ (1-f_d)*(X_BHf.*b_H).*V_fish +(1-
f_d)*(X_BAf.*b_A).*V_fish;%(b_A.*(1-f_d).*X_BHf.*V_fish +
b_H.*(1-f_d).*X_BAf.*V_fish + r_fecesinert)./V_fish;

%BIOREACTOR ODEs
    dS_Sb = (Q_f.*S_Sf)./V_fish -(Q_b.*S_Sb)./V_bioreactor +
(r_Hb.* (-1 ./ Y_H).*X_BHb.*V_bioreactor + r_hydrolysisb.*i_C_Xs
- r_Hb.*X_BHb.*V_bioreactor.*1.184)./V_bioreactor;
    dX_Sb = (Q_f.*X_Sf)./V_fish -(Q_b.*X_Sb)./V_bioreactor +
(b_H.*X_BHb.*V_bioreactor.*(f_d) +
b_A.*X_BAb.*V_bioreactor.*(f_d) - r_hydrolysisb)./V_bioreactor;
%+ rate(11).*(i_deg_feces - i_N_feces - i_P_feces); should it be
rate3 and rate4*(f_d - i_N,bio -i_P,bio)?
    dS_Ib = 0;
    dXi_b = (Q_f.*Xi_f)./V_fish -(Q_b.*Xi_b)./V_bioreactor + (1-
f_d).*(X_BHb.*b_H).*V_bioreactor +(1-
f_d).*(X_BAb.*b_A).*V_bioreactor;%(b_H.*(1-
f_d).*X_BHb.*V_bioreactor + b_A.*(1-
f_d).*X_BAb.*V_bioreactor)./V_bioreactor;%+X_Sb./5.666;
    dX_BHb = (Q_f.*X_BHf)./V_fish -(Q_b.*X_BHb)./V_bioreactor +
(r_Hb.*X_BHb.*V_bioreactor -
b_H.*X_BHb.*V_bioreactor)./V_bioreactor;
    dX_BAb =  (Q_f.*X_BAf)./V_fish -(Q_b.*X_BAb)./V_bioreactor +
(r_Ab.*X_BAb.*V_bioreactor -
b_A.*X_BAb.*V_bioreactor)./V_bioreactor;
    dS_NHb = (Q_f.*S_NHf)./V_fish -(Q_b.*S_NHb)./V_bioreactor +
(r_Hb.*(-i_N_bio).*X_BHb.*V_bioreactor + r_Ab.*(-
i_N_bio.*X_BAb.*V_bioreactor) -
r_Ab.*(1./Y_A).*X_BAb.*V_bioreactor +
k_a.*S_ONb.*X_Sb.*V_bioreactor)./V_bioreactor; %+ rate(12) -
rate(14).*(i_N_plant).*(S_NH./(S_NH+S_NO3));
    dS_NO3b =  (Q_f.*S_NO3f)./V_fish -
(Q_b.*S_NO3b)./V_bioreactor +
(r_Ab.*(1./Y_A).*X_BAb.*V_bioreactor)./V_bioreactor; %-
rate(14).*(i_N_plant).*(S_NO3./(S_NO3+S_NH));
    dS_ONb = (Q_f.*S_ONf)./V_fish -(Q_b.*S_ONb)./V_bioreactor +
(r_hydrolysisb.*i_N_Xs -
k_a.*S_ONb.*X_Sb.*V_bioreactor)./V_bioreactor;
    dS_O2b =  (r_Hb.*((-1-Y_H)./Y_H).*X_BHb.*V_bioreactor +
r_Ab.*((-4.57-Y_A)./Y_A).*X_BAb.*V_bioreactor +
r_aerationb)./V_bioreactor; % + rate(10).*((-1-Y_fish)./Y_fish) +
rate(16);
    %(Q_f.*S_O2f)./V_fish -(Q_b.*S_O2b)./V_bioreactor +
    dS_OPb =(Q_f.*S_OPf)./V_fish -(Q_b.*S_OPb)./V_bioreactor +
(r_hydrolysisb.*i_P_Xs - k_m.*S_OPb.*V_bioreactor)./V_bioreactor;
    dS_SRPb = (Q_f.*S_SRPf)./V_fish -(Q_b.*S_SRPb)./V_bioreactor
+ (r_Hb.*(-i_P_bio).*X_BHb.*V_bioreactor + r_Ab.*(-
```

```
i_P_bio).*X_BAb.*V_bioreactor +
k_m.*S_OPb.*V_bioreactor)./V_bioreactor; %-
rate(14).*(i_P_plant);
    dS_Kb = 0;
    dS_Mgb = 0;
    dS_Cab = 0;
    dCb = 0; %
    dNb = 0; %
    dPb = 0; %
%PLANT TANK ODEs
    dS_Sp = (Q_b.*S_Sb)./V_bioreactor- (Q_p.*S_Sp)./V_plant +
(r_Hp.* (-1 ./ Y_H).*X_BHp.*V_plant + r_hydrolysisp.*i_C_Xs-
r_Hp.*X_BHp.*V_plant.*1.184)./V_plant;
    dX_Sp = (Q_b.*X_Sb)./V_bioreactor-(Q_p.*X_Sp)./V_plant +
(b_H.*(f_d).*X_BHp.*V_plant + b_A.*(f_d).*X_BAp.*V_plant -
r_hydrolysisp)./V_plant; %+rate(11).*(i_deg_feces - i_N_feces -
i_P_feces)
    dS_Ip = 0;
    dXi_p = (Q_b.*Xi_b)./V_bioreactor-(Q_p.*Xi_p)./V_plant + (1-
f_d).*(X_BHp.*b_H).*V_plant +(1-
f_d).*(X_BAp.*b_A).*V_plant;%(b_H.*(1-f_d).*X_BHp.*V_plant +
b_A.*(1-f_d).*X_BAp.*V_plant)./V_plant;%X_Sp./5.666;
    dX_BHp =(Q_b.*X_BHb)./V_bioreactor-(Q_p.*X_BHp)./V_plant +
(r_Hp.*X_BHp.*V_plant - b_H.*X_BHp.*V_plant)./V_plant;
    dX_BAp = (Q_b.*X_BAb)./V_bioreactor-(Q_p.*X_BAp)./V_plant +
(r_Ap.*X_BAp.*V_plant - b_A.*X_BAp.*V_plant)./V_plant;
    dS_NHp = (Q_b.*S_NHb)./V_bioreactor-(Q_p.*S_NHp)./V_plant+
(r_Hp.*(-i_N_bio).*X_BHp.*V_plant)./V_plant + (r_Ap.*(-
i_N_bio.*X_BAp.*V_plant) -
r_Ap.*(1./Y_A).*X_BAp.*V_plant)./V_plant +
(k_a.*S_ONp.*X_Sp.*V_plant)./V_plant; %-
r_plant.*(i_N_plant).*(S_NHp./(S_NHp+S_NO3p)
    dS_NO3p =(Q_b.*S_NO3b)./V_bioreactor-(Q_p.*S_NO3p)./V_plant+
(r_Ap.*(1./Y_A).*X_BAp.*V_plant)./V_plant -
r_plant.*i_N_plant.*PlantDensityp.*A_plant./V_plant;
%.*(S_NO3p./(S_NO3p+S_NHp)
    dS_ONp = (Q_b.*S_ONb)./V_bioreactor-(Q_p.*S_ONp)./V_plant -
(k_a.*S_ONp.*X_Sp.*V_plant + r_hydrolysisp.*i_P_Xs)./V_plant;
    dS_O2p = (r_Hp.*((-1-Y_H)./Y_H).*X_BHp.*V_plant + r_Ap.*((-
4.57-Y_A)./Y_A).*X_BAp.*V_plant + r_aerationp)./V_plant; %
    dS_OPp = (Q_b.*S_OPb)./V_bioreactor-(Q_p.*S_OPp)./V_plant +
(r_hydrolysisp.*i_P_Xs - k_m.*S_OPp.*X_Sp.*V_plant)./V_plant;
    dS_SRPp =(Q_b.*S_SRPb)./V_bioreactor-(Q_p.*S_SRPp)./V_plant +
(r_Hp.*(-i_P_bio).*X_BHp.*V_plant + r_Ap.*(-
i_P_bio).*X_BAp.*V_plant + k_m.*S_OPp.*X_Sp.*V_plant -
r_plant.*(i_P_plant).*PlantDensityp.*A_plant)./V_plant;
    dS_Kp = (Q_b.*S_Kb)./V_bioreactor-(Q_p.*S_Kp)./V_plant -
(r_plant.*(i_K_plant).*PlantDensityp.*A_plant)./V_plant;
    dS_Mgp =(Q_b.*S_Mgb)./V_bioreactor-(Q_p.*S_Mgp)./V_plant -
(r_plant.*(i_Mg_plant).*PlantDensityp.*A_plant)./V_plant; %+
rate(11).*(i_Mg_feces)
```

```
    dS_Cap =(Q_b.*S_Cab)./V_bioreactor-(Q_p.*S_Cap)./V_plant -
(r_plant.*(i_Ca_plant).*PlantDensityp.*A_plant)./V_plant; %+
rate(11).*(i_Ca_feces)
  if t<=10
      r_plant = 0;
      dPp = 0;
  else
       r_plant = k_plant .* Y_plant .* (PPFD ./ (K_PPFD + PPFD
+(PPFD^2 ./ K_I_PPFD))) .* (S_NO3p ./ (K_NO3_plant +S_NO3p))
.*(P_CO2 ./ (K_PCO2_plant + P_CO2)).* (S_O2p ./(K_SO2_plant +
S_O2p))  .* (S_Kp./ (K_K_plant +S_Kp)) .* (S_Cap ./ (K_Ca_plant +
S_Cap)) .* (S_Mgp ./ (K_Mg_plant + S_Mgp)) .* (S_SRPp ./
(K_SRP_plant + S_SRPp));%.* (T_GH ./ (K_TGH_plant + T_GH +
(T_GH^2 ./ K_I_TGH_plant)));
       dPp = (r_plant./A_plant).*PlantDensityp.*A_plant -
(b_plant./A_plant).*PlantDensityp.*A_plant;
%(r_plant_harvest.*PlantDensityp.*A_plant)./V_plant;
  end
    dPlantDensityp = PlantDensityp.*A_plant.*(r_plant./V_plant) -
PlantDensityp.*A_plant.*(b_plant./V_plant) -
(PlantDensityp.*A_plant.*(r_plant./V_plant) -
PlantDensityp.*A_plant.*(b_plant./V_plant));%PlantDensityp.*A_pla
nt.*(r_plant_harvest./V_plant);
    dCp = 0; %
    dNp = 0; %
    dPplant = 0; %

%----Water----- (volume/time)
    dQ_P = (r_plant -
b_plant).*PlantDensityp.*A_plant.*i_water_plant; %(MASS/day of
water)
    dQ_H = (r_fishf.*Mfish - r_catabolism).*i_water_fish;
%MASS/day of water
    dQ_A = ((Q_ET + Q_evap).*10^6)./(p_GH.*omega_GH -
p_ext.*omega_ext);%(-(Q_ET+Q_evap)./(p_ext.*omega_ext -
p_GH.*omega_GH))./10^-6;%(-(Q_ET.*10^6
+Q_evap.*10^6)/(p_ext.*omega_ext - p_GH.*omega_GH);%older:
((Q_ET.*10^6)+(Q_evap.*10^6))./(omega_GH.*p_GH); %m3/d flow of
air
    dWater = Q_ET + Q_evap + Q_P +Q_H;%Q_A.*omega_ext.*p_ext -
Q_A.*omega_GH.*p_GH + Q_evap.*10^6 +Q_ET.*10^6; %MASS/day of
water.
%                   1    2      3     4      5      6      7
8      9
FISHBIOPLANTS =
[dS_Sf;dX_Sf;dS_If;dX_BHf;dX_BAf;dS_NHf;dS_NO3f;dS_ONf;dS_O2f;dC_
fishf;dS_OPf;dS_SRPf;dS_Kf;dS_Mgf;dS_Caf;dHf;dRf;dXi_f;dCf;dNf;dP
f;dS_Sb;dX_Sb;dS_Ib;dX_BHb;dX_BAb;dS_NHb;dS_NO3b;dS_ONb;dS_O2b;dS
_OPb;dS_SRPb;dS_Kb;dS_Mgb;dS_Cab;dXi_b;dCb;dNb;dPb;dS_Sp;dX_Sp;dS
_Ip;dX_BHp;dX_BAp;dS_NHp;dS_NO3p;dS_ONp;dS_O2p;dS_OPp;dS_SRPp;dS_
Kp;dS_Mgp;dS_Cap;dPlantDensityp;dPp;dXi_p;dCp;dNp;dPplant;dQ_P;dQ
_H;dQ_A;dWater;Q_ET];%r_food;r_fishf;r_fishresp;r_Hf;r_Af;r_Hresp
```

```
f;r_catabolism;r_gillN;r_fecesC;r_fecesN;r_fecesP;r_fecesK;r_fece
sCa;r_fecesMg;r_fecesinert;r_hydrolysisf;r_aeration;r_fish_harves
t;r_Hb;r_Ab;r_Hrespb;r_hydrolysisb;r_Hp;r_Ap;r_Hrespp;r_hydrolysi
sp;r_plant;r_plant_harvest;r_aerationb;r_aerationp];

end
```

A.1.2 Solver

```
clc
clear all
close all

%Numbers 1 -> 21
    SXf = [10 1 0 1 .1 6 5 5 8 68 5 5 5 5 5 0.1 30 1 11 16 11];
%Numbers 22 -> 39
    SXb = [10 1 0 1 .1 6 5 5 8 5 5 5 5 5 1 11 16 11];
%Numbers 40 -> 59
    SXp = [10 1 0 1 .1 6 5 5 8 5 5 5 5 5 101 1 1 11 16 11];
%Numbers 60 -> 61
    SXflow = [0.0001 0.0001 10 0.1 0.01];


    SX0 = [SXf SXb SXp SXflow];

[t,species]=ode15s(@Fishbioplants9g,0:1:25,SX0);

Parameters_only;

    S_Sf = species(end,1); %readily biodegradable substrate
    X_Sf = species(end,2); %slowly biodegradable (particulate)
substrate
   % S_If = species(end,3); %soluble inert material
    X_BHf = species(end,4); % active heterotropic biomass
    X_BAf = species(end,5); % active autotrophic biomass
    S_NHf = species(end,6) ; %ammonia nitrogen
    S_NO3f = species(end,7); % nitrate nitrogen
    S_ONf = species(end,8); % soluble organic nitrogen
    S_O2f = species(end,9); % oxygen concentration
    C_fishf = species(end,10); %fish
    S_OPf = species(end,11); %soluble organic phosphorus
    S_SRPf = species(end,12); %soluble reactive phosphorus
(orthophosphates)
    S_Kf = species(end,13); %free potassium ion
    S_Mgf = species(end,14); %free magnesium ion
    S_Caf = species(end,15); %free calcium ion
    Hf = species(end,16); %Fish harvest rate (g/day)
    Rf = species(end,17); %Fish feeding rate (g/day)
```

```
    Xi_f = species(end,18);
    Cf = species(end,19);
    Nf = species(end,20);
    Pf = species(end,21);
    S_Sb = species(end,22); %readily biodegradable substrate
    X_Sb = species(end,23); %slowly biodegradable (particulate)
substrate
    %S_Ib = species(end,24); %soluble inert material
    X_BHb = species(end,25); % active heterotropic biomass
    X_BAb = species(end,26); % active autotrophic biomass
    S_NHb = species(end,27) ; %ammonia nitrogen
    S_NO3b = species(end,28); % nitrate nitrogen
    S_ONb = species(end,29); % soluble organic nitrogen
    S_O2b = species(end,30); % oxygen concentration
    S_OPb = species(end,31); %soluble organic phosphorus
    S_SRPb = species(end,32); %soluble reactive phosphorus
(orthophosphates)
    S_Kb = species(end,33); %free potassium ion
    S_Mgb = species(end,34); %free magnesium ion
    S_Cab = species(end,35); %free calcium ion
    Xi_b = species(end,36);%inert material in the bioreactor
    Cb = species(end,37);
    Nb = species(end,38);
    Pb = species(end,39);
    S_Sp = species(end,40); %readily biodegradable substrate
    X_Sp = species(end,41); %slowly biodegradable (particulate)
substrate
    %S_Ip = species(end,42); %soluble inert material
    X_BHp = species(end,43); % active heterotropic biomass
    X_BAp = species(end,44); % active autotrophic biomass
    S_NHp = species(end,45) ; %ammonia nitrogen
    S_NO3p = species(end,46); % nitrate nitrogen
    S_ONp = species(end,47); % soluble organic nitrogen
    S_O2p = species(end,48); % oxygen concentration
    S_OPp = species(end,49); %soluble organic phosphorus
    S_SRPp = species(end,50); %soluble reactive phosphorus
    S_Kp = species(end,51); %free potassium ion
    S_Mgp = species(end,52); %free magnesium ion
    S_Cap = species(end,53); %free calcium ion
    PlantDensityp = species(end,54); %plant mass per area (g/m2)
    Pp = species(end,55); %plant harvest rate (g/day)
    Xi_p = species(end,56); %inert material in the plant tank
    Cp = species(end,57);
    Np = species(end,58);
    Pplant = species(end,59);
    Q_P = species(end,60);
    Q_H = species(end,61);
    Q_A = species(end,62);
    Water = species(end,63);
    Q_ET = species(end,64);

%----------------FISH TANK:--------------------
```

```
%Plot ammonium
plot(t,species(:,6),'linewidth',2)
hold on
%Plot nitrate
plot(t,species(:,7),'linewidth',2)
hold on
%plot XBH
plot(t,species(:,4),'-.','linewidth',2.5)
hold on
%plot XBA
plot(t,species(:,5),'-.','linewidth',1.5)
hold on
%plot SS
plot(t,species(:,1),'-.','linewidth',1.5)
hold on
%plot SON
plot(t,species(:,8),':','linewidth',2)
hold on
%plot SOP
plot(t,species(:,11),'linewidth',1)
hold on
%plot SRP
plot(t,species(:,12),':','linewidth',2)
hold on
%plot Oxygen
plot(t,species(:,9),':','linewidth',1.5)
hold on
%plot Xs
plot(t,species(:,2),'-.','linewidth',1)
hold on
%plot Ca
plot(t,species(:,15),'linewidth',1)
hold on
%plot K
plot(t,species(:,13),'linewidth',1)
hold on
%plot Mg
plot(t,species(:,14),'linewidth',1)
hold on
%plot fish GROWTH RATE (g fish/m3-d)
plot(t,species(:,16),'--','linewidth',2)
hold off

%Figure Format Setting
ylim([-1 11])
% xlim([0 5])
title('Fish Tank','Fontsize',12)
xlabel('Time (days)','Fontsize',16)
ylabel('Concentration (mg/L)','Fontsize',16)
legend('Ammonium-N','Nitrate-N','Heterotrophic
biomass','Autotrophic biomass','Soluble org. C','Soluble org.
N','Soluble org. P','Soluble reactive P','Dissolved
```

```
oxygen','Particulate
substrate','Calcium','Potassium','Magnesium','Fish growth')
%legend('Location','bestoutside')


%-------------BIOREACTOR:------------------------
figure
%Plot ammonium concentration
plot(t,species(:,27),'linewidth',2)
hold on
%plot nitrate
plot(t,species(:,28),'linewidth',2)
hold on
%plot XBH
plot(t,species(:,25),'-.','linewidth',2.5)
hold on
%plot XBA
plot(t,species(:,26),'-.','linewidth',1.5)
hold on
%plot Ss
plot(t,species(:,22),'-.','linewidth',1.5)
hold on
%plot SON
plot(t,species(:,29),':','linewidth',2)
hold on
%plot SOP
plot(t,species(:,31),'linewidth',1)
hold on
%plot SRP
plot(t,species(:,32),':','linewidth',2)
hold on
%plot oxygen
plot(t,species(:,30),':','linewidth',1)
hold on
%plot Xs
plot(t,species(:,23),'-.','linewidth',1)
%plot Ca
plot(t,species(:,35),'linewidth',1)
hold on
%plot K
plot(t,species(:,33),'linewidth',1)
hold on
%plot Mg
plot(t,species(:,34),'linewidth',1)
hold off

%Figure Format Setting
ylim([-1 9])
 % xlim([0 5])
 title('Bioreactor','Fontsize',12)
xlabel('Time (days)','Fontsize',16)
ylabel('Concentration (mg/L)','Fontsize',16)
```

```
%legend('Location','bestoutside')
legend('Ammonium-N','Nitrate-N','Heterotrophic
biomass','Autotrophic biomass','Soluble org. C','Soluble org.
N','Soluble org. P','Soluble reactive P','Dissolved
oxygen','Particulate
substrate','Calcium','Potassium','Magnesium')

%--------------PLANT TANK:-----------------------
figure
%Plot ammonium concentration
plot(t,species(:,45),'linewidth',2)
hold on
%plot nitrate
plot(t,species(:,46),'linewidth',2)
hold on
%plot XBH
plot(t,species(:,43),'-.','linewidth',2.5)
hold on
%plot XBA
plot(t,species(:,44),'-.','linewidth',2)
hold on
%plot Ss
plot(t,species(:,40),'-.','linewidth',2)
hold on
%plot SON
plot(t,species(:,47),':','linewidth',2)
hold on
%plot SOP
plot(t,species(:,49),'linewidth',1)
hold on
%plot SRP
plot(t,species(:,50),':','linewidth',2)
hold on
%plot oxygen
plot(t,species(:,48),'-.','linewidth',1)
hold on
%plot Xs
plot(t,species(:,41),'-.','linewidth',1)
%plot Ca
plot(t,species(:,53),'linewidth',1)
hold on
%plot K
plot(t,species(:,51),'linewidth',1)
hold on
%plot Mg
plot(t,species(:,52),'linewidth',1)
hold on
%plot plant GROWTH RATE (g/m2-d)
plot(t,species(:,55),'--','linewidth',2)
hold off

%Figure Format Setting
```

```
ylim([-1 30])
% xlim([0 5])
title('Plant Tank','Fontsize',12)
xlabel('Time (days)','Fontsize',16)
ylabel('Concentration (mg/L)','Fontsize',16)
legend('Ammonium-N','Nitrate-N','Heterotrophic
biomass','Autotrophic biomass','Soluble org. C','Soluble org.
N','Soluble org. P','Soluble reactive P','Dissolved
oxygen','Particulate
substrate','Calcium','Potassium','Magnesium','Plant growth
(g/m2)')
%legend('Location','bestoutside')

%-------------Water flows------------------T
figure
% %Plot Q_P
plot(t,species(:,60),'linewidth',1)
 hold on
% %plot Q_H
plot(t,species(:,61),'linewidth',1)
hold on
%plot Q_ET
%plot(t,Q_ET)
hold off

title('Water losses','Fontsize',12)
xlabel('Time (days)','Fontsize',16)
ylabel('Water lost (m^3)','Fontsize',16)
legend('Plant harvest water','Fish harvest water')
legend('Location','bestoutside')
```

REFERENCES

1. Use, L. & Change, L. 5. Agriculture. 1–49 (2015).

2. Rabalais, N. N., Turner, R. E. & Wiseman, W. J. Gulf of Mexico Hypoxia, A.K.A. 'The Dead Zone'. *Annu. Rev. Ecol. Syst.* **33,** 235–263 (2002).

3. Michael Beman, J., Arrigo, K. R. & Matson, P. A. Agricultural runoff fuels large phytoplankton blooms in vulnerable areas of the ocean. *Nature* **434,** 211–214 (2005).

4. Famiglietti, J. S. *et al.* Satellites measure recent rates of groundwater depletion in California's Central Valley. *Geophys. Res. Lett.* **38,** 2–5 (2011).

5. California's Central Valley. *USGS* (2017). Available at: https://ca.water.usgs.gov/projects/central-valley/about-central-valley.html.

6. Van Ginkel, S. W., Igou, T. & Chen, Y. Energy, water and nutrient impacts of California-grown vegetables compared to controlled environmental agriculture systems in Atlanta, GA. *Resour. Conserv. Recycl.* **122,** 319–325 (2017).

7. Weber, C. L. & Matthews, H. S. Food-miles and the relative climate impacts of food choices in the United States. *Environ. Sci. Technol.* **42,** 3508–3513 (2008).

8. Lee, S. K. & Kader, A. A. Preharvest and postharvest factors influencing vitamin C content of horticultural crops. *Postharvest Biol. Technol.* **20,** 207–220 (2000).

9. Canals, L. M. I., Muñoz, I., Hospido, A., Plassmann, K. & McLaren, S. Life Cycle Assessment (LCA) of domestic vs. imported vegetables. Case studies on broccoli, salad crops and green beans. *United Kingdom, Cent. Environ. Strateg. Univ. Surrey* 46 (2008). doi:1464-8083

10. Naylor, R. L. *et al.* Effect of aquaculture on world fish supplies. *Nature* **405,** 1017–24 (2000).

11. Jasinski, S. 2008 Minerals Yearbook: Potash. 1–10 (2010).

12. Déry, P. & Anderson, B. Peak phosphorus Prospect of a Phosphorus Peak. (2007).

13. Gellings, C. & Parmenter, K. Energy efficiency in fertilizer production and use. *Encycl. Life Support Syst.* 1–15 (2004).

14. Sarker, P. K. *et al.* Towards sustainable aquafeeds: Complete substitution of fish oil with marine microalga Schizochytrium sp. improves growth and fatty acid deposition in juvenile Nile tilapia (Oreochromis niloticus). *PLoS One* **11,** 1–17 (2016).

15. Diener, S., Zurbrugg, C. & Tockner, K. Conversion of organic material by black soldier fly larvae: establishing optimal feeding rates. *Waste Manag. Res.* **27,** 603–610 (2009).

16. Ghosh, C. Integrated vermi-pisciculture -- an alternative option for recycling of solid municipal waste in rural India. *Bioresour. Technol.* **93,** 71–75 (2004).

17. Crittenden, J. C. Water for Everything and the Transformative Technologies to Improve Water Sustainability. *Natl. Water Res. Inst. Clarke Prize Lect.* 1–23 (2015).

18. Endut, A., Jusoh, A., Ali, N., Wan Nik, W. B. & Hassan, A. A study on the optimal hydraulic loading rate and plant ratios in recirculation aquaponic system. *Bioresour. Technol.* **101,** 1511–1517 (2010).

19. Rakocy, J. E., Masser, M. P. & Losordo, T. M. Recirculating aquaculture tank production systems: Aquaponics- integrating fish and plant culture. *SRAC Publ. - South. Reg. Aquac. Cent.* 16 (2006). doi:454

20. Henze, M., Gujer, W., Mino, T. & van Loosdrecht, M. C. M. Activated Sludge Models ASM1, ASM2, ASM2d and ASM3. *IWA Publ.* 121 (2000). doi:10.1007/s13398-014-0173-7.2

21. monod kinetics.PDF.

22. Liu, W. & Lee, C. Practical Identification Analysis of Haldane Kinetic Parameters Describing Phenol Biodegradation in Batch Operations. *J. Environ. Enginering Manag.* **17,** 71–80 (2007).

23. Smith, H. W. *Excretion of Ammonia and Urea by the Gills of Fish.* (1929).

24. Wirén, N. Von, Gazzarrini, S. & Frommer, W. Regulation of mineral nitrogen uptake in plants. *Plant and Soil* **196,** 191–197 (1997).

25. Abbasi, H., Vasileva, V. & Lu, X. The Influence of the Ratio of Nitrate to Ammonium Nitrogen on Nitrogen Removal in the Economical Growth of Vegetation in Hybrid Constructed Wetlands. *Environments* **4,** 24 (2017).

26. Van Der Boon, J., Steenhuizen, J. W. & Steingrover, E. G. Growth and nitrate concentration of lettuce as affected by total nitrogen and chloride concentration, NH4/N03 ratio and temperature of the recirculating nutrient solution. *J. Hortic. Sci.* **65,** 309–321 (1990).

27. Graber, A. & Junge, R. Aquaponic Systems: Nutrient recycling from fish wastewater by vegetable production. *Desalination* **246,** 147–156 (2009).

28. Dampin, N., Tarnchalanukit, W., Chunkao, K. & Maleewong, M. Fish Growth Model for Nile Tilapia (Oreochromis niloticus) in Wastewater Oxidation Pond, Thailand. *Procedia Environ. Sci.* **13,** 513–524 (2012).

29. Yi, Y. a Bioenergetics Growth Model for Nile Tilapia (Oreochromis Niloticus) in a Cage-Gum-Pond Integrated System. *Thail. Spec. Top. Res. 2* 113–128 (1988).

30. Brechner, M. & Both, A. J. Hydroponic Lettuce Handbook. *Cornell Univ. CEA Progr.* 48 (1996).

31. Azaza, M. S., Dhraïef, M. N. & Kraïem, M. M. Effects of water temperature on growth and sex ratio of juvenile Nile tilapia Oreochromis niloticus (Linnaeus) reared in geothermal waters in southern Tunisia. *J. Therm. Biol.* **33,** 98–105 (2008).

32. Benjamin, M. *Water Chemistry*. (McGraw Hill, 2001).

33. DeBlasio, S. L., Luesse, D. L. & Hangarter, R. P. A plant-specific protein essential for blue-light-induced chloroplast movements. *Plant Physiol.* **139,** 101–114 (2005).

34. Holding, D. R. & Streich, A. M. Plant Growth Processes: Transpiration , photosynthesis, and respiration. 1–10 (2013).

35. Caporn S.J.M. The effects of oxides of nitrogen and carbon dioxide enrichment on the growth and content of ribulose-1,5-bisphosphate carboxylase oxygenase and nitrite reductase in glasshouse lettuce (Lactuca sativa L.). *New Phytol.* **111,** 473–481 (1989).

36. Farnsworth, R. K. & Thompson, E. S. *NOAA Technical Report NWS 34, Mean Monthly, Seasonal, and Annual Pan Evaporation for the United States*. (1982).

37. Allen, R. G., Pereira, L. S., Raes, D. & Smith, M. *Crop evapotranspiration - Guidelines for computing crop water requirements*. (1998).

38. Tsai, Y. P. & Wu, W. M. Estimating Biomass of Heterotrophic and Autotrophic Bacteria by our Batch Tests. *Environ. Technol.* **26,** 601–614 (2005).

39. Ryzhakov, A. V., Kukkonen, N. A. & Lozovik, P. A. Determination of the rate of ammonification and nitrification in natural water by kinetic method. *Water Resour.* **37,** 70–74 (2010).

40. Gönenç, I. E. & Wolflin, J. P. *Coastal Lagoons: ecosystem processes and modeling for sustainable use and development*. (CRC Press, 2005).

41. Lane, S. L. & Fay, R. G. Safety in field activities: U.S. Geological Survey Techniques of Water-Resources Investigations. in *9* 28–33 (1997).

42. Rittmann, B. E. & McCarty, P. L. *Environmental Biotechnology: Principles and Applications.* (McGraw Hill, 2001).

43. Wang, X. *et al.* Chemical composition and release rate of waste discharge from an Atlantic salmon farm with an evaluation of IMTA feasibility. *Aquac. Environ. Interact.* **4,** 147–162 (2013).

44. Naylor, S. J., Moccia, R. D. & Durant, G. M. The Chemical Composition of Settleable Solid Fish Waste (Manure) from Commercial Rainbow Trout Farms in Ontario, Canada. *N. Am. J. Aquac.* **61,** 21–26 (1999).

45. Anderson, T., de Villiers, D. & Timmons, M. Growth and Tissue Elemental Composition Response of Butterhead Lettuce (Lactuca sativa, cv. Flandria) to Hydroponic and Aquaponic Conditions. *Horticulturae* **3,** 43 (2017).

46. Murray, J. & Burt, J. R. The Composition of Fish. *FAO - Food and Agriculture Organization of the United Nations* (2001). Available at: http://www.fao.org/wairdocs/tan/x5916e/x5916e00.htm#Contents.

47. Adefemi, O. S. Chemical composition of Tilapia mosambis fish from major dams in Ekiti-State , Nigeria. *African J. Food Sci.* **5,** 550–554 (2011).

48. Pavlou, G. C., Ehaliotis, C. D. & Kavvadias, V. A. Effect of organic and inorganic fertilizers applied during successive crop seasons on growth and nitrate accumulation in lettuce. *Sci. Hortic. (Amsterdam).* **111,** 319–325 (2007).

49. Van Henten, E. J. & Van Straten, G. Sensitivity Analysis of a Dynamic Growth Model of Lettuce. *J. Agric. Eng. Res.* **59,** 19–31 (1994).

50. Lasa, B. *et al.* Effects of low and high levels of magnesium on the response of sunflower plants grown with ammonium and nitrate. *Plant Soil* **225,** 167–174 (2000).

51. Tilapia Feeding: a guide by Lakeway Tilapia. *Lakeway Tilapia* (2017). Available at: https://lakewaytilapia.com/Tilapia-Feeding-Guide.php.

52. Robinson, E. H., Li, M. H. & Brunson, M. W. Feeding Catfish in Commercial Ponds. *SRAC Publ.* **181,** (1998).

53. Al Hafedh, Y. S. Effects of dietary protein on growth and body composition of Nile tilapia, Oreochromis niloticus L. *Aquac. Res.* **30,** 385–393 (1999).

54. Filipe, C. D. . & Leslie Grady Jr., C. P. *Biological Wastewater Treatment*. (CRC Press, 1998).

55. Nile Tilapia - Nutritional requirements. *Food and Agriculture Organization of the United Nations* (2017). Available at: http://www.fao.org/fishery/affris/species-profiles/nile-tilapia/nutritional-requirements/en/. (Accessed: 11th September 2017)

56. Nwanna, L. C., Kühlwein, H. & Schwarz, F. J. Phosphorus requirement of common carp (Cyprinus carpio L) based on growth and mineralization. *Aquac. Res.* **41,** 401–410 (2010).

57. Boyd, C. E. & Green, B. W. Dry Matter, Ash, and Elemental Composition of Pond-Culture Tilapia Oreochromis aureus and O. niloticus. *J. World Aquac. Soc.* **29,** 125–

128 (1998).

58. Kalantarian, S. H., Rafiee, G. H., Farhangi, M. & Mojazi Amiri, B. Effect of Different Levels of Dietary Calcium and Potassium on Growth Indices, Biochemical Composition and Some Whole Body Minerals in Rainbow Trout (Oncorhynchus mykiss) Fingerlings. *Aquac. Res. Dev.* **4,** (2013).

59. Bijvelds, M. J. C., Flik, G., Kolar, Z. I. & Wendelaar Bonga, S. E. Uptake, distribution and excretion of magnesium inOreochromis mossambicus: dependence on magnesium in diet and water. *Fish Physiol. Biochem.* **15,** 287–298 (1996).

60. Atlanta, GA, USA. *Latlong.net* (2017). Available at: https://www.latlong.net/place/atlanta-ga-usa-6566.html. (Accessed: 11th August 2017)

61. Urieli, I. Water Vapor Saturation Data (Tsat, Pg) and the Psychrometric Chart. *Ohio University Russ College of Engineering and Technology* (2017). Available at: https://www.ohio.edu/mechanical/thermo/property_tables/H2O/psychro_chart.html. (Accessed: 11th August 2017)

62. Love, D. C., Uhl, M. S. & Genello, L. Energy and water use of a small-scale raft aquaponics system in Baltimore, Maryland, United States. *Aquac. Eng.* **68,** 19–27 (2015).

63. Yogev, U., Barnes, A. & Gross, A. Nutrients and energy balance analysis for a conceptual model of a three loops off grid, aquaponics. *Water (Switzerland)* **8,** (2016).

64. Jige, S. *et al.* Performance of decentralized algal energy system in urban contexts.